What is the intersection of the line $5x+2y=48$ and line $3x+2y=32$.

A test has 60 questions. The true/false questions are worth 2 points each and long answer are worth 5 points each for a total of 210 possible marks. How many true/false questions and long answer questions are there.

10 buckets and 10 cups can hold 80 litres of water. However, 5 buckets minus 5 cups of water holds 30 litres of water. What is the volume of the bucket and cup.

Someone has 15 coins in nickels and quarters. If that person has 3 more quarters than nickels, how many quarters and nickels does he/she have.

At a store the price of object A and object B is 54$. If object A is 8$ more than 3 times object B what is the price of object A and B.

100 tickets were sold for an event. Tickets A cost 5$ while ticket B cost 3$. The event made 422$ from the sale of tickets A and B. Who many tickets A and B were sold?

It takes a airplane 5.0 hours to travel from Toronto to Vancouver (3,364 km). It take the same airplane 4.5 hours to travel from Vancouver to Toronto due to the wind. What is the average velocity of the airplane and the average velocity of the wind?

Find the equation of the 1st order polynomial (equation of the line) that passes through the points (-1,5) and (1,2).

Find the equation of the 2nd order polynomial (parabola) that passes through the points (-1,5), (1,2) and (2,4).

Find the partial fraction decomposition of the following equation: $y = \frac{5x+1}{(x-1)(x+1)(x+2)}$

A company sells three products: A, B and C. Given that:

2A's, 3B's and 1C cost 17$ and

1A, 5B's and 2C's cost 20$.

Also, if someone returns A (i.e. gives back 1A) and buys 1B and 1C it costs 1$.

What are the prices of A, B and C.

Find the equation for the ages of two people given the ratio of their ages. The two people are N years apart in age.

What is the intersection of the line 5x+2y=48 and line 3x+2y=32.

$$\begin{array}{rl} 5x + 2y = 48 & (1) \\ -(3x + 2y = 32) & (2) \\ \hline 2x + 0y = 16 & \end{array}$$

$$\boxed{x = 8}$$

sub into (1)

$$5x + 2y = 48$$

$$y = \frac{48 - 5x}{2}$$

$$y = \frac{48 - 5(8)}{2} = 4$$

$$\boxed{y = 4}$$

The intersectin is at point (8,4).

A test has 60 questions. The true/false questions are worth 2 points each and long answer are worth 5 points each for a total of 210 possible marks. How many true/false questions and long answer questions are there.

x = # of true/false questions

y = # of long answer questons

$$x + y = 60 \qquad (1)$$

$$2x + 5y = 210 \qquad (2)$$

rearrange equ. (1)

$$x = 60 - y \qquad (1a)$$

sub into (2)

$$2(60-y) + 5y = 210$$

$$120 - 2y + 5y = 210$$

$$3y = 90$$

$$\boxed{y = 30}$$

sub into (1a)

$$x = 60 - (30)$$

$$\boxed{x = 30}$$

There are 30 true/false and 30 long answer questions

10 buckets and 10 cups can hold 80 litres of water. However, 5 buckets minus 5 cups of water holds 30 litres of water. What is the volume of the bucket and cup.

x = volume of bucket.

y = volume of cup.

$$10x + 10y = 80 \qquad (1)$$

$$5x - 5y = 30 \qquad (2)$$

Multiply equ. (2) by 2.

$$\begin{array}{r} 10x + 10y = 80 \\ +(10x - 10y = 60) \\ \hline 20x + 0y = 140 \end{array}$$

$$\boxed{x = 7}$$

Sub into equ. (1)

$$y = 8 - x$$

$$= 8 - 7 = 1$$

$$\boxed{y = 1}$$

The volume of the bucket is 7 litres and the volume of the cup is 1 litre.

Someone has 15 coins in nickels and quarters. If that person has 3 more quarters than nickels, how many quarters and nickels does he/she have.

x = # of nickels (worth 5¢ each)

y = # of quarters (worth 25¢ each)

$$x + y = 15 \qquad (1)$$

$$x + 3 = y \qquad (2)$$

rearrange equ. (2)

$$x = y - 3 \qquad (2a)$$

sub into equ. (1)

$$(y-3) + y = 15$$

$$y - 3 + y = 15$$

$$2y = 18$$

$$\boxed{y = 9}$$

sub into (2a)

$$x = (9) - 3 = 6$$

$$\boxed{x = 6}$$

They have 6 nickels and 9 quarters

At a store the price of object A and object B is 54$. If object A is 8$ more than 3 times object B what is the price of object A and B.

A = price of object A

B = price of object B

$A + B = 54 \quad (1)$

$A = 3B + 8 \quad (2)$

sub equ. (2) into (1)

$A + B = 54$

$(3B + 8) + B = 54$

$3B + 8 + B = 54$

$4B = 46$

$\boxed{B = 11.5}$

sub into equ. (2)

$A = 3B + 8$

$= 3(11.5) + 8$

$= 42.5$

$\boxed{A = 42.5}$

Object A costs \$42.50 and object B costs \$11.50.

100 tickets were sold for an event. Tickets A cost 5$ while ticket B cost 3$. The event made 422$ from the sale of tickets A and B. Who many tickets A and B were sold?

A = # of ticket A sold

B = # of ticket B sold.

$$A + B = 100 \qquad (1)$$

$$5A + 3B = 422 \qquad (2)$$

rearrange eqn. (1)

$$A = 100 - B \qquad (1a)$$

sub (1a) into (2)

$$5(100-B) + 3B = 422$$

$$500 - 5B + 3B = 422$$

$$-2B = -78$$

$$\boxed{B = 39}$$

sub into equation (1a)

$$A = 100 - B = 100 - 39 = 61$$

$$\boxed{A = 61}$$

There were 61 ticket A's sold and 39 ticket B's sold.

It takes a airplane 5.0 hours to travel from Toronto to Vancouver (3,364 km). It take the same airplane 4.5 hours to travel from Vancouver to Toronto due to the wind. What is the average velocity of the airplane and the average velocity of the wind?

V_p = velocity of plane (average)

V_w = velocity of wind (average)

$$V = \frac{\text{distance}}{\text{time}} \quad \text{or} \quad \boxed{(\text{time})(v) = \text{distance}}$$

from Toronto to Vancouver: $(5)(V_p - V_w) = 3364$ (1)

from Vancouver to Toronto: $(4.5)(V_p + V_w) = 3364$ (2)

$$5V_p - 5V_w = 3364$$
$$4.5V_p + 4.5V_w = 3364$$

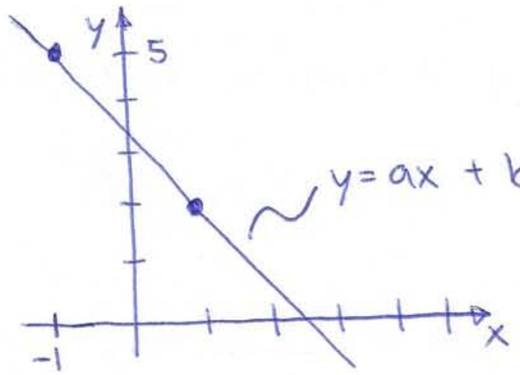
$$\begin{bmatrix} 5 & -5 & 3364 \\ 4.5 & 4.5 & 3364 \end{bmatrix}$$

$$\begin{array}{r} R_1/5 \\ R_2 - 4.5R_1^* \end{array} \begin{bmatrix} 1 & -1 & 672.8 \\ 0 & 9 & 336.4 \end{bmatrix}$$

$$\begin{array}{r} R_1 + R_2^* \\ R_2/9 \end{array} \begin{bmatrix} 1 & 0 & 710 \\ 0 & 1 & 37 \end{bmatrix}$$

Velocity of plane is 710 km/h and velocity of wind is 37 km/h

Find the equation of the 1st order polynomial (equation of the line) that passes through the points (-1,5) and (1,2).



point 1: $5 = a(-1) + b$

point 2: $2 = a(1) + b$

equations:

$$a + b = 2$$
$$-a + b = 5$$

$$\begin{bmatrix} 1 & 1 & 2 \\ -1 & 1 & 5 \end{bmatrix}$$

$$\begin{array}{c} R_1 \\ R_2 + R_1 \end{array} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 7 \end{bmatrix}$$

$$\begin{array}{c} R_1 - R_2^* \\ R_2/2 \end{array} \begin{bmatrix} 1 & 0 & -1.5 \\ 0 & 1 & 3.5 \end{bmatrix}$$

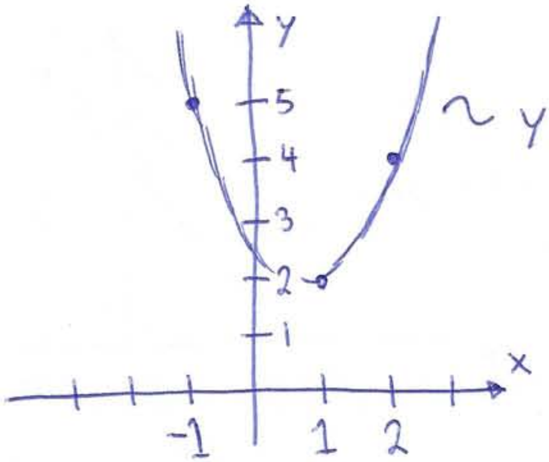
$a = -1.5$

$b = 3.5$

equation of line:

$$\boxed{y = -1.5x + 3.5}$$

Find the equation of the 2nd order polynomial (parabola) that passes through the points (-1,5), (1,2) and (2,4).



$\sim y = ax^2 + bx + c$

point 1 $(-1,5)$: $a(-1)^2 + b(-1) + c = 5$
point 2 $(1,2)$: $a(1)^2 + b(1) + c = 2$
point 3 $(2,4)$: $a(2)^2 + b(2) + c = 4$

Equations:

$$a - b + c = 5$$
$$a + b + c = 2$$
$$4a + 2b + c = 4$$

$$\begin{bmatrix} 1 & -1 & 1 & 5 \\ 1 & 1 & 1 & 2 \\ 4 & 2 & 1 & 4 \end{bmatrix}$$

$$\begin{array}{c} R_1 \\ R_2 - R_1 \\ R_3 - 4R_1 \end{array} \begin{bmatrix} 1 & -1 & 1 & 5 \\ 0 & 2 & 0 & -3 \\ 0 & 6 & -3 & -16 \end{bmatrix}$$

$$\begin{array}{c} R_1 \\ R_2/2 \\ R_3 - 6R_2 \end{array} \begin{bmatrix} 1 & -1 & 1 & 5 \\ 0 & 1 & 0 & -1.5 \\ 0 & 0 & -3 & -7 \end{bmatrix}$$

$$\begin{array}{c} R_1 - R_3^* \\ R_2 \\ R_3/-3 \end{array} \begin{bmatrix} 1 & -1 & 0 & 2\frac{2}{3} \\ 0 & 1 & 0 & -1.5 \\ 0 & 0 & 1 & \frac{7}{3} \end{bmatrix}$$

$$\begin{array}{c} R_1 + R_2 \\ \\ \\ \end{array} \begin{bmatrix} 1 & 0 & 0 & \frac{7}{6} \\ 0 & 1 & 0 & -1.5 \\ 0 & 0 & 1 & \frac{7}{3} \end{bmatrix}$$

equation:
$$\boxed{1.1\dot{6}x^2 - 1.5x + 2.\overline{3} = y}$$

Find the partial fraction decomposition of the following equation: $y = \dfrac{5x+1}{(x-1)(x+1)(x+2)}$

$$\frac{5x+1}{(x-1)(x+1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x+2}$$

$$\frac{5x+1}{\cancel{(x-1)(x+1)(x+2)}} = \frac{A(x+1)(x+2) + B(x-1)(x+2) + C(x-1)(x+1)}{\cancel{(x-1)(x+1)(x+2)}}$$

$$5x+1 = Ax^2 + A3x + A2 + Bx^2 + Bx - B2 + Cx^2 - C$$

$$x^2: \quad 0 = A + B + C$$
$$x^1: \quad 5 = 3A + B$$
$$x^0: \quad 1 = 2A - 2B - C$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 3 & 1 & 0 & 5 \\ 2 & -2 & -1 & 1 \end{bmatrix}$$

$$\begin{array}{r} R_1 \\ R_2 - 3R_1 \\ R_3 - 2R_1 \end{array} \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & -2 & -3 & 5 \\ 0 & -4 & -3 & 1 \end{bmatrix}$$

$$\begin{array}{r} R_1 \\ R_2/-2 \\ R_3 + 4R_2^* \end{array} \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1.5 & -2.5 \\ 0 & 0 & 3 & -9 \end{bmatrix}$$

$$\begin{array}{r} R_1 - R_3^* \\ R_2 - 1.5R_3^* \\ R_3/3 \end{array} \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3 \end{bmatrix}$$

$$\begin{array}{r} R_1 - R_2 \\ R_2 \\ R_3 \end{array} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3 \end{bmatrix}$$

$$\boxed{A = 1,\ B = 2,\ C = -3}$$

$$\boxed{= \frac{1}{x-1} + \frac{2}{x+1} - \frac{3}{x+2}}$$

A company sells three products: A, B and C. Given that:

2A's, 3B's and 1C cost 17$ and

1A, 5B's and 2C's cost 20$.

Also, if someone returns A (i.e. gives back 1A) and buys 1B and 1C it costs 1$.
What are the prices of A, B and C.

$$2A + 3B + C = 17$$
$$A + 5B + 2C = 20$$
$$-A + B + C = 1$$

$$\begin{bmatrix} 2 & 3 & 1 & 17 \\ 1 & 5 & 2 & 20 \\ -1 & 1 & 1 & 1 \end{bmatrix}$$

$$\begin{array}{r} R_1/2 \\ R_2 - R_1^* \\ R_3 + R_1^* \end{array} \begin{bmatrix} 1 & 1.5 & 0.5 & 8.5 \\ 0 & 3.5 & 1.5 & 11.5 \\ 0 & 2.5 & 1.5 & 9.5 \end{bmatrix}$$

$$\begin{array}{r} R_1 \\ R_2/3.5 \\ R_3 - 2.5R_2^* \end{array} \begin{bmatrix} 1 & 1.5 & 0.5 & 8.5 \\ 0 & 1 & 3/7 & 23/7 \\ 0 & 0 & 3/7 & 9/7 \end{bmatrix}$$

$$\begin{array}{r} R_1 - 0.5R_3^* \\ R_2 - \frac{3}{7}R_3^* \\ R_3/(3/7) \end{array} \begin{bmatrix} 1 & 1.5 & 0 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$\begin{array}{r} R_1 - 1.5R_2 \\ R_2 \\ R_3 \end{array} \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$A = 4, \; B = 2, \; C = 3$$

Object A costs 4$ and object B costs 2$ and object C costs 3$.

Find the equation for the ages of two people given the ratio of their ages. The two people are N years apart in age.

a = age of older person.

b = age of younger person.

$$a = b + N \qquad (1)$$

$$\frac{a}{b} = r = \text{ratio of ages.} \qquad (2)$$

sub equ. (1) into equ. (2)

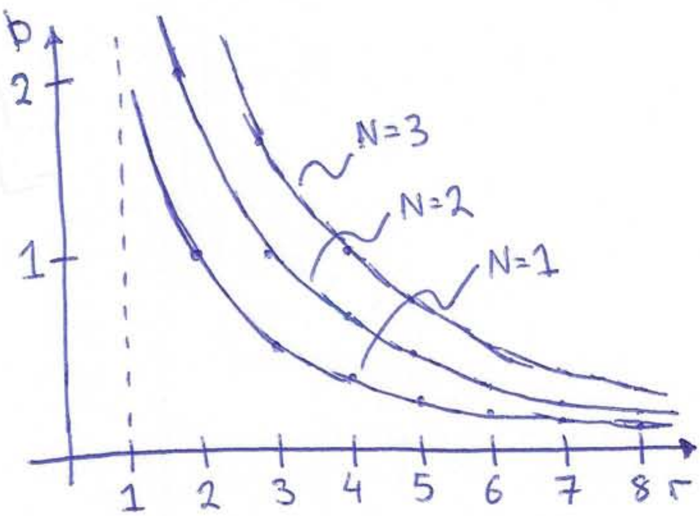
$$\frac{b+N}{b} = r$$

$$b + N = br$$

$$N = b(r-1)$$

$$\boxed{b = \frac{N}{(r-1)}}$$

$$b = \frac{N}{r-1}$$



sub into equ. (1)

$$a = \left(\frac{N}{r-1}\right) + N$$

$$= \frac{N}{r-1} + \frac{N(r-1)}{r-1}$$

$$= \frac{\cancel{N} + Nr - \cancel{N}}{r-1}$$

$$\boxed{a = \frac{Nr}{r-1}}$$