Instructor:	Frank Secretain
Course:	Math 101
Assessment:	Final Exam
Time allowed:	110 minutes
Devices allowed:	Pencil, pen, eraser, calculator
Notes from instructor:	Be neat. Show your work where needed. Box final answers.
Marks allocated:	8 questions worth 35 marks
Percentage of final grade:	20% of final grade

Formula Sheet

Order of Operations

$$ac + bc = c(a + b)$$

exponents

$$a^n a^m = a^{n+m}$$

$$(a^n)^m = a^{nm}$$

$$(ab)^n = a^n b^n$$

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

radicals

$$a^{\frac{n}{m}} = \sqrt[m]{a^n}$$

Relative Velocity

$$\vec{v}_{\frac{A}{C}} = \vec{v}_{\frac{A}{B}} + \vec{v}_{\frac{B}{C}}$$

Linear equations (Cramer's rule)

$$x_i = \frac{det(A_i)}{det(A)}$$

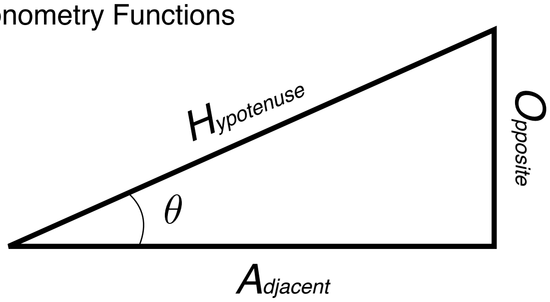
Forms of a 2nd order polynomial

$$y = ax^2 + bx + c$$

$$y = a(x - h)^2 + k$$

$$y = (x - m)(x - n)$$

Trigonometry Functions



$$sin(\theta) = \frac{O}{H} \qquad sin^{-1}\left(\frac{O}{H}\right) = \theta$$

$$cos(\theta) = \frac{A}{H} \qquad cos^{-1}\left(\frac{A}{H}\right) = \theta$$

$$tan(\theta) = \frac{O}{A} \qquad tan^{-1}\left(\frac{O}{A}\right) = \theta$$

Pythagoras Theorem

$$H^2 = O^2 + A^2$$

Unit Conversions

angles

$$2\pi = 6.28 \text{ rad} = 360^o$$

mass

$$1 \text{ kg} = 2.2 \text{ lbs.}$$

lengths

$$1 \text{ mile} = 1.6 \text{ km}$$

$$1 \text{ inch} = 2.54 \text{ cm}$$

$$1 \text{ m} = 3.3 \text{ ft}$$

volumes

$$1 \text{ gallon} = 3.78 \text{ Litres}$$

(3 marks) Match the “type of number” with the best “example number”. Draw a line to match the “type of number” to the “example number” to indicate your answer.

whole	$\frac{3}{4}$
rational	$\sqrt{2}$
real	2

(2 marks) Solve the each expression and keep the correct number of significant digits.

3.45+(0.0034)(1284.49)

20.64+(5.6)(85.6)

(2 marks) Convert each number into scientific notation.

0.04930

3090

(3 marks) Convert each of the numbers to the stated units.

$$\frac{3\pi}{2} \to \text{degrees}$$

$$6.5\ \frac{\text{gallons}}{\text{sec}} \to \frac{\text{Litres}}{\text{hour}}$$

$$30\ \frac{\text{m}}{\text{s}} \to \frac{\text{miles}}{\text{hour}}$$

(5 marks) You ran 320 m North, 120 m at 30^0 South of East and an unknown distance. If you ended up only 35 m West of where you started what was the unknown distance? You do not need to determine the angle, just the distance.

(3 marks each) Solve for x in the following equations

$$\frac{x-1}{x+1}+1=0$$

$$\frac{4(x-1)^2-2\big[x+2(x^2-1)\big]}{x-1}=1$$

$$\frac{m_1 x + m_2 a + m_3 b}{m_1 m_2 m_3} = \delta$$

$$\frac{(\eta - 1)^2 \gamma \beta - x}{x - 1} + 1 = \lambda$$

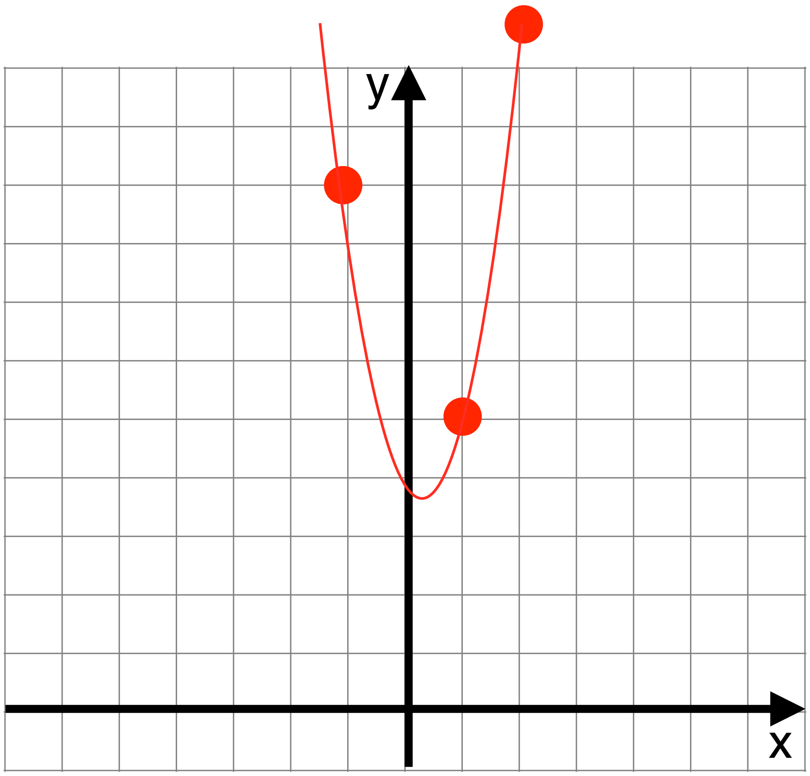
(5 marks) The perimeter of a rectangular is 612 meters. The length is 33 meters more than twice the width. Find the dimension of the garden.

(3 marks) Plot the function in the space provided, ensure to label all relevant axises.

$$y(x) = 2x^2 - 4x + 5$$

(2 marks) Bonus:

Find the equation of the parabola that passes through the points (–1, 9), (1, 5), and (2, 12)



(3 marks) Match the “type of number” with the best “example number”. Draw a line to match the “type of number” to the “example number” to indicate your answer.

whole	$\frac{3}{4}$
rational	$\sqrt{2}$
real	2

(2 marks) Solve the each expression and keep the correct number of significant digits.

3.45+(0.0034)(1284.49)

3.45 + 4.367266

7.817266 => $\boxed{7.8}$

20.64+(5.6)(85.6)

20.64 + 479.36

500 => $\boxed{5.0 \times 10^{2}}$

(2 marks) Convert each number into scientific notation.

0.04930

$\boxed{4.930 \times 10^{-2}}$

3090

$\boxed{3.09 \times 10^{3}}$

(3 marks) Convert each of the numbers to the stated units.

$$\frac{3\pi}{2} \rightarrow \text{degrees}$$

$$\frac{3\pi}{2}\left(\frac{360^\circ}{2\pi}\right) = 270^\circ$$

$$\boxed{270^\circ}$$

$$6.5\ \frac{\text{gallons}}{\text{sec}} \rightarrow \frac{\text{Litres}}{\text{hour}}$$

$$6.5\ \frac{\text{gallons}}{\text{sec}}\left(\frac{3.78\ \text{L}}{1\ \text{gallon}}\right)\left(\frac{60\ \text{sec}}{1\ \text{min}}\right)\left(\frac{60\ \text{min}}{1\ \text{hour}}\right) = 88452\ \frac{\text{L}}{\text{hour}}$$

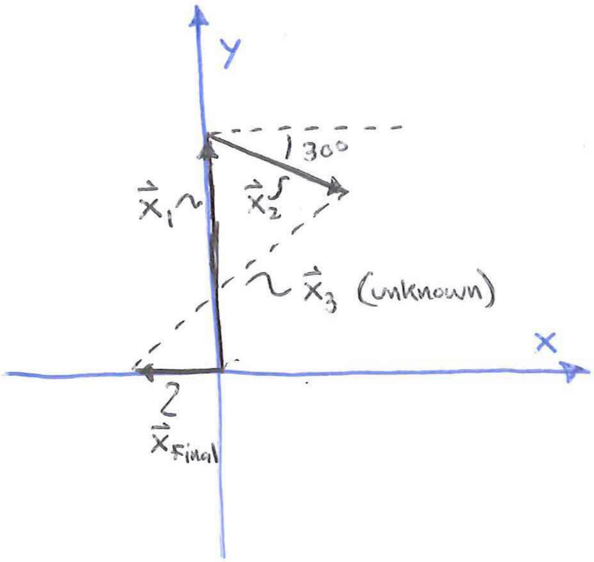
$$\boxed{88000\ \frac{\text{L}}{\text{hour}}}$$

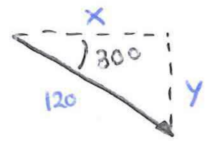
$$30\ \frac{\text{m}}{\text{s}} \rightarrow \frac{\text{miles}}{\text{hour}}$$

$$30\ \frac{\text{m}}{\text{s}}\left(\frac{1\ \text{km}}{1000\ \text{m}}\right)\left(\frac{1\ \text{mile}}{1.6\ \text{km}}\right)\left(\frac{60\ \text{sec}}{1\ \text{min}}\right)\left(\frac{60\ \text{min}}{1\ \text{hour}}\right) = 67.5\ \frac{\text{miles}}{\text{h}}$$

$$\boxed{70\ \frac{\text{miles}}{\text{hour}}}$$

(5 marks) You ran 320 m North, 120 m at 30⁰ South of East and an unknown distance. If you ended up only 35 m West of where you started what was the unknown distance? You do not need to determine the angle, just the distance.





$x = 120 \cos(30) = 103.92$

$y = 120 \sin(30) = 60$

$$\boxed{\vec{X}_{Final} = \vec{X}_1 + \vec{X}_2 + \vec{X}_3}$$

so $\vec{X}_3 = \vec{X}_{Final} - \vec{X}_1 - \vec{X}_2$

$$
\begin{aligned}
\vec{X}_{Final} &= -35\hat{x} + 0\hat{y} \\
-\vec{X}_1 &= 0\hat{x} - 320\hat{y} \\
-\vec{X}_2 &= -103.92\hat{x} + 60\hat{y} \\
\hline
\vec{X}_3 &= -138.92\hat{x} - 260\hat{y}
\end{aligned}
$$

$|\vec{X}_3| = \sqrt{(-138.92)^2 + (-260)^2}$

$= 294.79$ m

$$\boxed{|\vec{X}_3| = 295 \text{ m}}$$

(3 marks each) Solve for x in the following equations

$$\frac{x-1}{x+1}+1=0$$

$$\frac{x-1}{x+1} = -1$$

$$x - \cancel{1} = -x - \cancel{1}$$

$$2x = 0$$

$$x = 0$$

$$\boxed{x = 0}$$

$$\frac{4(x-1)^2 - 2\left[x + 2(x^2-1)\right]}{x-1} = 1$$

$$4(x^2-2x+1) - 2[x + 2x^2 - 2] = x - 1$$

$$\cancel{4x^2} - 8x + 4 - 2x - \cancel{4x^2} + 4 = x - 1$$

$$-11x = -9$$

$$x = \frac{9}{11}$$

$$\boxed{x = \frac{9}{11} \cong 0.818}$$

$$\frac{m_1 x + m_2 a + m_3 b}{m_1 m_2 m_3} = \delta$$

$$m_1 x + m_2 a + m_3 b = \delta m_1 m_2 m_3$$

$$m_1 x = \delta m_1 m_2 m_3 - m_2 a - m_3 b$$

$$x = \frac{\delta m_1 m_2 m_3 - m_2 a - m_3 b}{m_1}$$

$$\boxed{x = \frac{\delta m_1 m_2 m_3 - m_2 a - m_3 b}{m_1}}$$

$$\frac{(\eta - 1)^2 \gamma \beta - x}{x - 1} + 1 = \lambda$$

$$(\eta - 1)^2 \gamma \beta \cancel{- x} \cancel{+ x} - 1 = \lambda (x - 1)$$

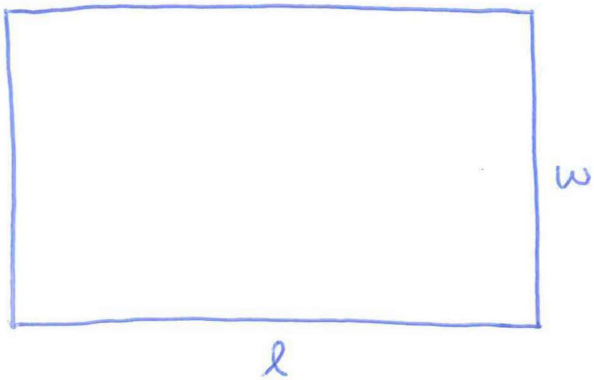
$$(\eta - 1)^2 \gamma \beta - 1 = \lambda x - \lambda$$

$$(\eta - 1)^2 \gamma \beta - 1 + \lambda = \lambda x$$

$$x = \frac{(\eta - 1)^2 \gamma \beta - 1 + \lambda}{\lambda}$$

$$\boxed{x = \frac{(\eta - 1)^2 \gamma \beta + \lambda - 1}{\lambda}}$$

(5 marks) The perimeter of a rectangular is 612 meters. The length is 33 meters more than twice the width. Find the dimension of the garden.



let:

l = lenght of rectangle

w = width of rectangle

so:

$$612 = 2l + 2w \quad (1)$$

$$l = 2w + 33 \quad (2)$$

sub (2) into (1):

$$612 = 2[2w + 33] + 2w$$

$$612 = 4w + 66 + 2w$$

$$546 = 6w$$

$$w = 91$$

sub back into (2)

$$l = 2[91] + 33$$

$$l = 215$$

so

$l = 215$ m

$w = 91$ m

(3 marks) Plot the function in the space provided, ensure to label all relevant axises.

$$y(x) = 2x^2 - 4x + 5$$

$$2x^2 - 4x + 5 = a(x-h)^2 + k$$

$$2x^2 - 4x + 5 = ax^2 - 2ahx + ah^2 + k$$

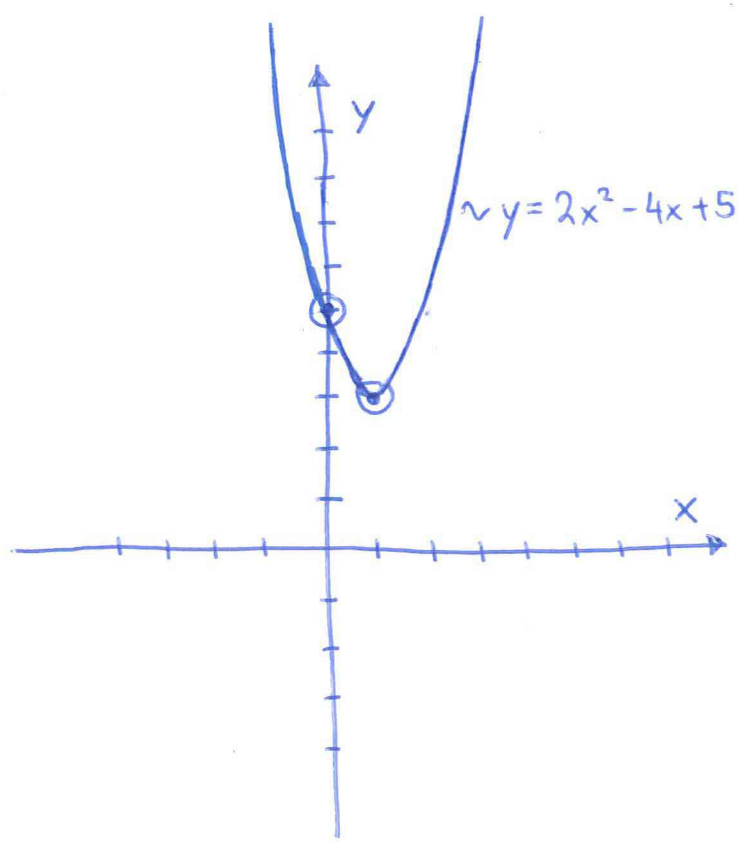
x^2: $\quad 2 = a \quad \Rightarrow a = 2$

x^1: $\quad -4 = -2ah \quad \Rightarrow h = 1$

x^0: $\quad 5 = ah^2 + k \quad \Rightarrow k = 3$

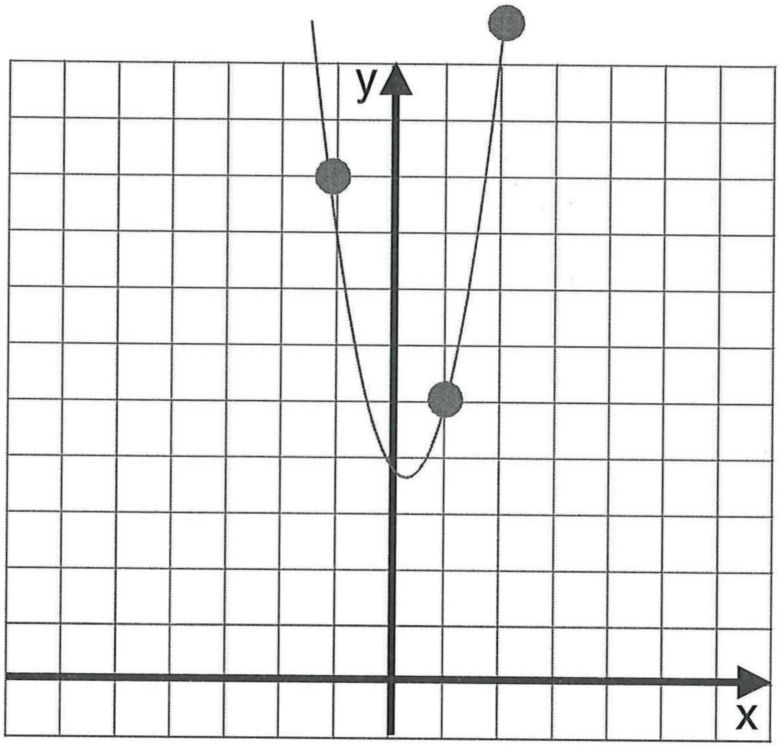
so

$$y(x) = 2(x-1)^2 + 3$$



(2 marks) Bonus:

Find the equation of the parabola that passes through the points (–1, 9), (1, 5), and (2, 12)



$y = ax^2 + bx + c$

sub points:

$9 = a - b + c$ (1)

$5 = a + b + c$ (2)

$12 = 4a + 2b + c$ (3)

solve for a in (1)

$a = 9 + b - c$ (1a)

sub (1a) into (2) & (3)

$5 = [9 + b - c] + b + c$

$5 = 9 + 2b$

$b = -2$ (2a)

$12 = 4[9 + b - c] + 2b + c$

$12 = 36 + 4b - 4c + 2b + c$

$-24 = 6b - 3c$ (3a)

Solve for c in (3a) & sub (2a)

$c = \frac{-24 - 6b}{-3}$

$c = \frac{-24 - 6[-2]}{-3}$

$c = 4$ (3b)

sub (2a) & (3b) into (1a)

$a = 9 + [-2] - [4]$

$a = 3$

so

$y = 3x^2 - 2x + 4$