Instructor:	Frank Secretain
Course:	Math 101
Assessment:	Test 1
Time allowed:	110 minutes
Devices allowed:	Pencil, pen, eraser, calculator
Notes from instructor:	Be neat. Show your work where needed. Box final answers.
Marks allocated:	6 questions worth 30 marks
Percentage of final grade:	20% of final grade

Formula Sheet

Order of Operations

$$ac + bc = c(a + b)$$

exponents

$$a^n a^m = a^{n+m}$$

$$(a^n)^m = a^{nm}$$

$$(ab)^n = a^n b^n$$

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

radicals

$$a^{\frac{n}{m}} = \sqrt[m]{a^n}$$

Relative Velocity

$$\vec{v}_{\frac{A}{C}} = \vec{v}_{\frac{A}{B}} + \vec{v}_{\frac{B}{C}}$$

Linear equations (Cramer's rule)

$$x_i = \frac{det(A_i)}{det(A)}$$

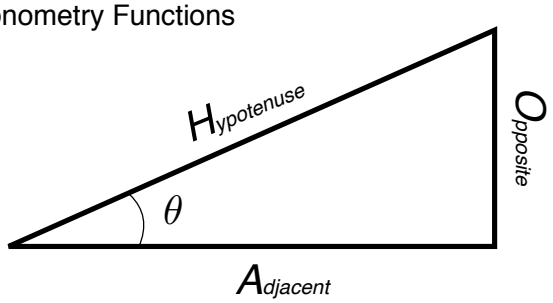
Forms of a 2nd order polynomial

$$y = ax^2 + bx + c$$

$$y = a(x - h)^2 + k$$

$$y = (x - m)(x - n)$$

Trigonometry Functions



$$sin(\theta) = \frac{O}{H} \qquad sin^{-1}\left(\frac{O}{H}\right) = \theta$$

$$cos(\theta) = \frac{A}{H} \qquad cos^{-1}\left(\frac{A}{H}\right) = \theta$$

$$tan(\theta) = \frac{O}{A} \qquad tan^{-1}\left(\frac{O}{A}\right) = \theta$$

Pythagoras Theorem

$$H^2 = O^2 + A^2$$

Unit Conversions

angles

$$2\pi = 6.28 \text{ rad} = 360^o$$

mass

$$1 \text{ kg} = 2.2 \text{ lbs.}$$

lengths

$$1 \text{ mile} = 1.6 \text{ km}$$

$$1 \text{ inch} = 2.54 \text{ cm}$$

$$1 \text{ m} = 3.3 \text{ ft}$$

volumes

$$1 \text{ gallon} = 3.78 \text{ Litres}$$

(5 marks) Match the “type of number” with the best “example number”. Draw a line to match the “type of number” to the “example number” to indicate your answer.

natural	0
whole	$\frac{3}{4}$
rational	$\sqrt{2}$
imaginary	$\sqrt{-2}$
real	2

(3 marks) Solve the each expression and keep the correct number of significant digits.

1.234+0.01483

1.234+(1.2)(1200)

102.3+(2.34)(8.40)

(2 marks) Convert each number into scientific notation.

0.0034520

120000

(2 marks) Convert each number in scientific notation to a decimal number.

7.563×10^{3}

2.3×10^{-3}

(3 marks) Convert each of the numbers to the stated units.

$$67^{o} \rightarrow \text{radians}$$

$$80 \ \frac{\text{miles}}{\text{hour}} \rightarrow \frac{\text{m}}{\text{s}}$$

$$1.2 \ \frac{\text{cm}^3}{\text{s}} \rightarrow \frac{\text{ft}^3}{\text{hour}}$$

(5 marks) You run 100 m North, 30 m East, 60 m at 30^0 South of East and 40 m 20^0 West of North. How far are you from where you started?

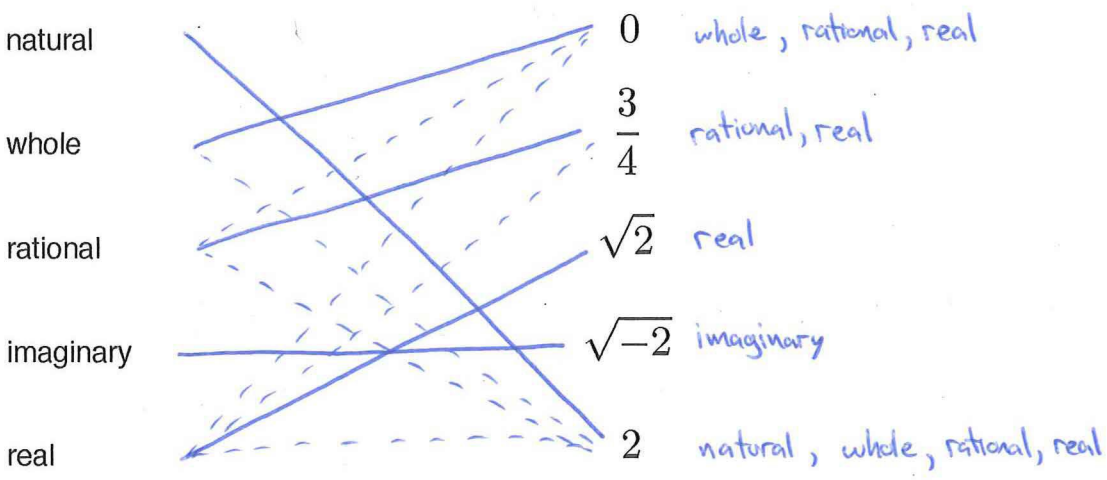
(5 marks) Your on a boat! You are walking to the front of the boat with a velocity of 2 m/s relative to the boat. The water is moving relative to the ground at a velocity of 5 m/s and an angle of 30^0 East of North relative to the ground. Your friend (who is sitting on shore) observes you moving away from him at 12 m/s at an angle of 40^0 North of West. How fast and in what direction is the boat moving relative to the water?

(5 marks) Solve for x in the following equations

$$4x^2 - 4(x+1)^2 = 4$$

$$\frac{2x-a}{b-1} + \Phi = 0$$

(5 marks) Match the “type of number” with the best “example number”. Draw a line to match the “type of number” to the “example number” to indicate your answer.



(3 marks) Solve the each expression and keep the correct number of significant digits.

1.234+0.01483

$= 1.234 + 0.01483$ (+3, +5)

$= 1.2483$ (+3)

$= 1.249$

1.234+(1.2)(1200)

$= 1.234 + \frac{(1.2)(1200)}{2}$

$= 1.234 + \frac{1440}{2}$ (+3, −2)

$= 1441.234$

$= 1400$

102.3+(2.34)(8.40)

$= 102.3 + \frac{(2.34)(8.40)}{3}$

$= 102.3 + \frac{19.656}{3}$ (+1, +1)

$= 121.956$ (+1)

$= 122.0$

(2 marks) Convert each number into scientific notation.

0.0034520

$= 3.4520 \times 10^{-3}$

120000

$= 1.2 \times 10^{5}$

(2 marks) Convert each number in scientific notation to a decimal number.

7.563×10^{3}

$= 7563$

2.3×10^{-3}

$= 0.0023$

(3 marks) Convert each of the numbers to the stated units.

$67^{o} \rightarrow$ radians

$$67^{\circ}\left(\frac{2\pi}{360^{\circ}}\right) = \frac{67\pi}{180} = 0.372\pi = 1.17 \text{ rad}$$

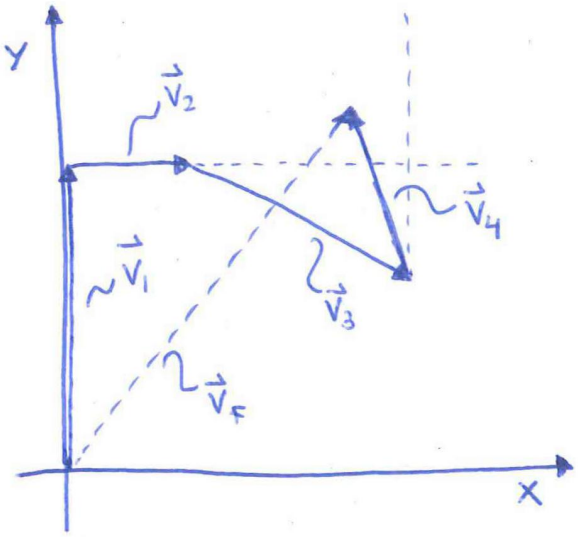
$$80 \frac{\text{miles}}{\text{hour}} \rightarrow \frac{\text{m}}{\text{s}}$$

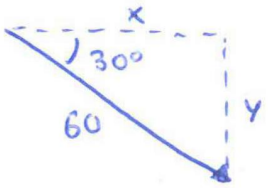
$$80 \frac{\text{miles}}{\text{hour}}\left(\frac{1.6 \text{ km}}{1 \text{ mile}}\right)\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ hour}}{60 \text{ min}}\right)\left(\frac{1 \text{ min}}{60 \text{ sec}}\right) = 35.6 \frac{\text{m}}{\text{s}}$$

$$1.2 \frac{\text{cm}^3}{\text{s}} \rightarrow \frac{\text{ft}^3}{\text{hour}}$$

$$1.2 \frac{\text{cm}^3}{\text{s}}\left(\frac{1 \text{ m}}{100 \text{ cm}}\right)^3\left(\frac{3.3 \text{ ft}}{1 \text{ m}}\right)^3\left(\frac{60 \text{ s}}{1 \text{ min}}\right)\left(\frac{60 \text{ min}}{1 \text{ hour}}\right) = 0.155 \frac{\text{ft}^3}{\text{hour}}$$

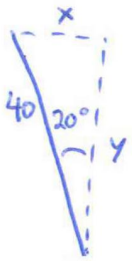
(5 marks) You run 100 m North, 30 m East, 60 m at 30^0 South of East and 40 m 20^0 West of North. How far are you from where you started?





$x = 60\cos(30)$
$= 51.96$

$y = 60\sin(30)$
$= 30$



$x = 40\sin(20)$
$= 13.68$

$y = 40\cos(20)$
$= 37.59$

$$\vec{V}_F = \vec{V}_1 + \vec{V}_2 + \vec{V}_3 + \vec{V}_4$$

$\vec{V}_1 = 0\hat{x} + 100\hat{y}$

$\vec{V}_2 = 30\hat{x} + 0\hat{y}$

$\vec{V}_3 = 51.96\hat{x} - 30\hat{y}$

$\vec{V}_4 = -13.68\hat{x} + 37.59\hat{y}$

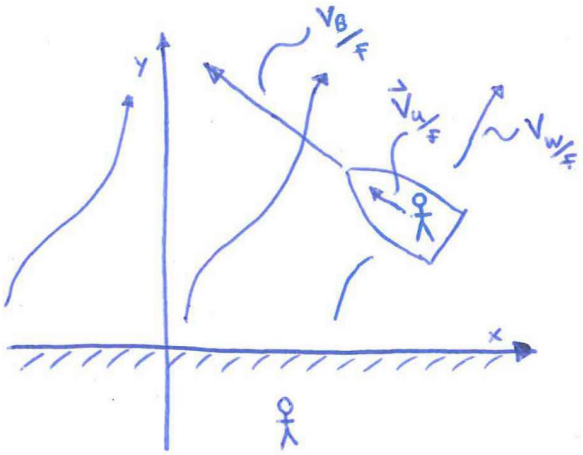
$\vec{V}_F = 68.28\hat{x} + 107.59\hat{y}$

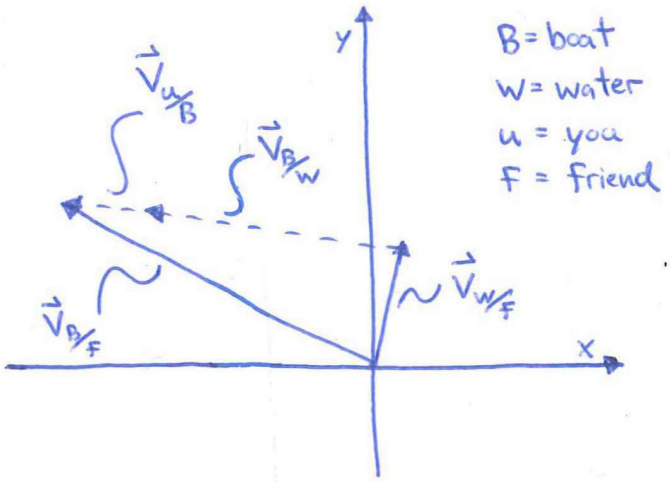
$|\vec{V}_F| = \sqrt{(68.28)^2 + (107.59)^2}$

$= 127.43$

$|\vec{V}_F| = 127\ m$

(5 marks) Your on a boat! You are walking to the front of the boat with a velocity of 2 m/s relative to the boat. The water is moving relative to the ground at a velocity of 5 m/s and an angle of 30^0 East of North relative to the ground. Your friend (who is sitting on shore) observes you moving away from him at 12 m/s at an angle of 40^0 North of West. How fast and in what direction is the boat moving relative to the water?



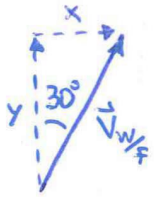


$$\vec{V}_{u/F} = \vec{V}_{u/B} + \vec{V}_{B/w} + \vec{V}_{w/F}$$

$$\vec{V}_{u/F} = \vec{V}_{u/w} + \vec{V}_{w/F}$$

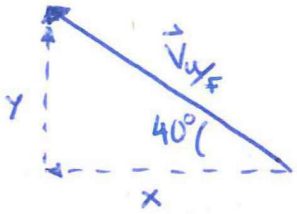
$$\vec{V}_{u/w} = \vec{V}_{u/F} - \vec{V}_{w/F}$$

$$\boxed{\vec{V}_{u/w} = \vec{V}_{u/F} + \left(-\vec{V}_{w/F}\right)}$$



$$x = 5\sin(30) = 2.5$$

$$y = 5\cos(30) = 4.33$$



$$x = 12\cos(40) = 9.19$$

$$y = 12\sin(40) = 7.71$$

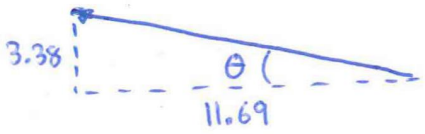
$$\vec{V}_{u/F} = -9.19\,\hat{x} + 7.71\,\hat{y}$$

$$-\vec{V}_{w/F} = -2.5\,\hat{x} - 4.33\,\hat{y}$$

$$\vec{V}_{u/w} = -11.69 + 3.38\,\hat{y}$$

$$\left|\vec{V}_{u/w}\right| = \sqrt{(-11.69)^2 + (3.38)^2}$$

$$= 12.2$$



$$\theta = \tan^{-1}\left(\frac{3.38}{11.69}\right) = 16.1^\circ$$

$$\left|\vec{V}_{B/w}\right| = \left|\vec{V}_{u/w}\right| - \left|\vec{V}_{u/B}\right| = 12.2 - 2 = 10.2 \text{ m/s}$$

(5 marks) Solve for x in the following equations

$$4x^2 - 4(x+1)^2 = 4$$

$4x^2 - 4(x^2 + 2x + 1) = 4$

$4x^2 - 4x^2 - 8x - 4 = 4$

$-8x = 8$

$\boxed{x = -1}$

$$\frac{2x - a}{b - 1} + \Phi = 0$$

$\frac{2x - a}{b-1} = -\Phi$

$2x - a = (-\Phi)(b-1)$

$2x = \Phi - \Phi b + a$

$\boxed{x = \frac{\Phi - \Phi b + a}{2} = \frac{\Phi(1-b) + a}{2} = \frac{a - \Phi(b-1)}{2}}$