Instructor:	Frank Secretain
Course:	Math 101
Assessment:	Test 2
Time allowed:	110 minutes
Devices allowed:	Pencil, pen, eraser, calculator
Notes from instructor:	Be neat. Show your work where needed. Box final answers.
Marks allocated:	5 questions worth 30 marks
Percentage of final grade:	20% of final grade

Formula Sheet

Order of Operations

$$ac + bc = c(a + b)$$

exponents

$$a^n a^m = a^{n+m}$$

$$(a^n)^m = a^{nm}$$

$$(ab)^n = a^n b^n$$

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

radicals

$$a^{\frac{n}{m}} = \sqrt[m]{a^n}$$

Relative Velocity

$$\vec{v}_{\frac{A}{C}} = \vec{v}_{\frac{A}{B}} + \vec{v}_{\frac{B}{C}}$$

Linear equations (Cramer's rule)

$$x_i = \frac{det(A_i)}{det(A)}$$

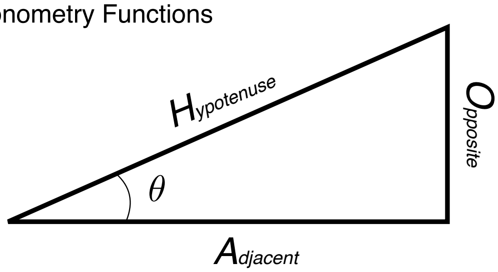
Forms of a 2nd order polynomial

$$y = ax^2 + bx + c$$

$$y = a(x - h)^2 + k$$

$$y = (x - m)(x - n)$$

Trigonometry Functions



$$sin(\theta) = \frac{O}{H} \qquad sin^{-1}\left(\frac{O}{H}\right) = \theta$$

$$cos(\theta) = \frac{A}{H} \qquad cos^{-1}\left(\frac{A}{H}\right) = \theta$$

$$tan(\theta) = \frac{O}{A} \qquad tan^{-1}\left(\frac{O}{A}\right) = \theta$$

Pythagoras Theorem

$$H^2 = O^2 + A^2$$

Unit Conversions

angles

$$2\pi = 6.28 \text{ rad} = 360^o$$

mass

$$1 \text{ kg} = 2.2 \text{ lbs.}$$

lengths

$$1 \text{ mile} = 1.6 \text{ km}$$

$$1 \text{ inch} = 2.54 \text{ cm}$$

$$1 \text{ m} = 3.3 \text{ ft}$$

volumes

$$1 \text{ gallon} = 3.78 \text{ Litres}$$

Simplify the expression (5 marks):

$$\frac{2x(x-2)^2+x}{x+1}+1$$

Solve for x in the expressions (3 marks each):

$$\frac{2x(x+1)-x^2}{x}=4x-1$$

$$8x^2(x+1)-2\big(4x^3+2x(2x-4)\big)=1$$

$$I_o\alpha_o = \big(mg\cos(\theta)\big)\left(\frac{l}{2}\right) - \big(x\sin(\theta)\big)\big(l\big)$$

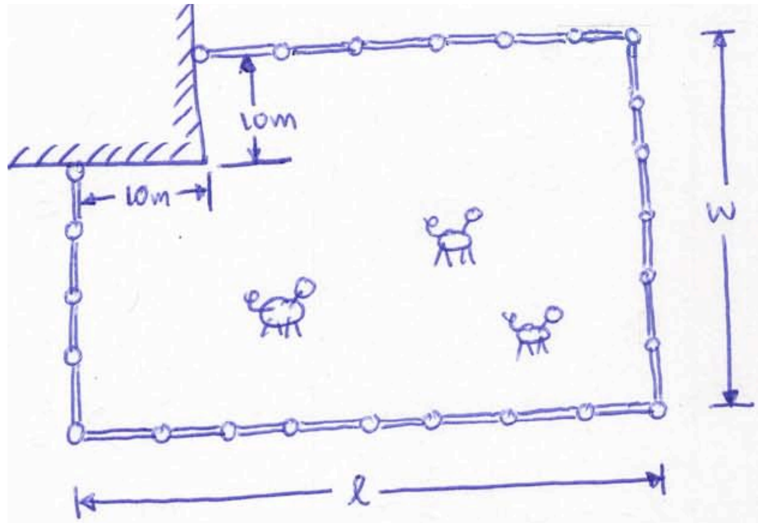
$$\Phi + \frac{2\gamma[4\Delta - \alpha]^2(\mu - 1)}{4ax} = \alpha^2$$

Solve the system of linear equations (3 marks):

$$3x - 19 + 5y = 0$$
$$2x + 5y - 10 = 2 + 2y$$

(5 marks) Timmy is 21 and 1/2 years younger than his good friend Bob. However, in 10 years, Bob will be twice as old as Timmy. What are their current ages?

(5 marks) You are provided with 84.3 m of fencing and are required to build a fence as shown in the figure. Note, that a 10 m by 10 m corner does not need to be fenced. Further, the length should be 1.62 times larger than the width. What are the dimensions of the fence?



Simplify the expression (5 marks):

$$\frac{2x(x-2)^2+x}{x+1}+1$$

$$\frac{2x(x^2-4x+4)+x}{x+1}+1$$

$$\frac{2x^3-8x^2+8x+x}{x+1}+1$$

$$\frac{2x^3-8x^2+9x}{x+1}+1$$

modified long division

$$\frac{2x^3-8x^2+9x}{x+1} = ax^2+bx+c+\frac{d}{x+1}$$

$$2x^3-8x^2+9x = ax^3+ax+bx^2+bx+cx+c+d$$

x^3: $2 = a \Rightarrow a = 2$

x^2: $-8 = a + b \Rightarrow b = -10$

x^1: $9 = b + c \Rightarrow c = 19$

x^0: $0 = c + d \Rightarrow d = -19$

so

$$\frac{2x^3-8x^2+9x}{x+1}+1 = 2x^2-10x+19-\frac{19}{x+1}+1$$

$$\boxed{2x^2-10x+20-\frac{19}{x+1}}$$

Simplify the expression (5 marks):

$$\frac{2x(x-2)^2+x}{x+1}+1$$

$$\frac{2x(x^2-4x+4)+x}{x+1}+1$$

$$\frac{2x^3-8x^2+8x+x}{x+1}+1$$

$$\frac{2x^3-8x^2+9x}{x+1}+1$$

long division

$$\begin{array}{r} 2x^2 - 10x + 19 \\ x+1 \overline{\big) 2x^3 - 8x^2 + 9x + 0} \\ -(2x^3 + 2x^2) \\ \hline 0 \quad -10x^2 + 9x \\ -(-10x^2 - 10x) \\ \hline 0 \quad 19x + 0 \\ -(19x + 19) \\ \hline 0 \quad -19 \end{array}$$

so

$$\frac{2x^3-8x^2+9x}{x+1}+1 = 2x^2-10x+19-\frac{19}{x+1}+1$$

$$\boxed{2x^2-10x+20-\frac{19}{x+1}}$$

Solve for x in the expressions (3 marks each):

$$\frac{2x(x+1) - x^2}{x} = 4x - 1$$

$$\frac{2x^2 + 2x - x^2}{x} = 4x - 1$$

$$\frac{x^2 + 2x}{x} = 4x - 1$$

$$x + 2 = 4x - 1$$

$$3 = 3x$$

$$\boxed{x = 1}$$

note:

$$x^2 + 2x = 4x^2 - x$$

$x = 0$ would also be a solution, however does not satisfy initial equation i.e. $\frac{0}{0}$

$$8x^2(x+1) - 2\big(4x^3 + 2x(2x-4)\big) = 1$$

$$8x^3 + 8x^2 - 2(4x^3 + 4x^2 - 8x) = 1$$

$$8x^3 + 8x^2 - 8x^3 - 8x^2 + 16x = 1$$

$$\boxed{x = \frac{1}{16}}$$

$$I_o\alpha_o = (mg\cos(\theta))\left(\frac{l}{2}\right) - (x\sin(\theta))(l)$$

$$x(\sin\theta)(l) = (mg\cos\theta)\left(\frac{l}{2}\right) - I_o\alpha_o$$

$$x = \frac{\frac{l}{2}mg\cos\theta - I_o\alpha_o}{l\sin\theta}$$

$$x = \frac{l\,mg\cos\theta - 2I_o\alpha_o}{2l\sin\theta}$$

$$\Phi + \frac{2\gamma[4\Delta - \alpha]^2(\mu - 1)}{4ax} = \alpha^2$$

$$\frac{2\gamma[4\Delta - \alpha]^2(\nu - 1)}{4ax} = \alpha^2 - \Phi$$

$$2\gamma[4\Delta - \alpha]^2(\nu - 1) = 4ax(\alpha^2 - \Phi)$$

$$\boxed{x = \frac{2\gamma[4\Delta - \alpha]^2(\nu - 1)}{4a(\alpha^2 - \Phi)}}$$

Solve the system of linear equations (3 marks):

$$3x - 19 + 5y = 0$$
$$2x + 5y - 10 = 2 + 2y$$

$$3x + 5y = 19 \quad (1)$$

$$2x + 3y = 12 \quad (2)$$

solve for x in (2)

$$x = \frac{12 - 3y}{2} \quad (2a)$$

sub (2a) into (1)

$$3\left[\frac{12 - 3y}{2}\right] + 5y = 19$$

$$3\left[12 - 3y\right] + 10y = 38$$

$$36 - 9y + 10y = 38$$

$$\boxed{y = 2}$$

sub backinto (2a)

$$x = \frac{12 - 3[2]}{2} = \frac{6}{2} = 3$$

$$\boxed{x = 3}$$

(5 marks) Timmy is 21 and 1/2 years younger than his good friend Bob. However, in 10 years, Bob will be twice as old as Timmy. What are their current ages?

let

T = Timmy's current age

B = Bob's current age

so

$$T + 21.5 = B \qquad (1)$$

$$\frac{B+10}{T+10} = 2 \quad \Rightarrow \quad B + 10 = 2(T+10) \qquad (2)$$

sub (1) into (2)

$$[T + 21.5] + 10 = 2(T + 10)$$

$$T + 31.5 = 2T + 20$$

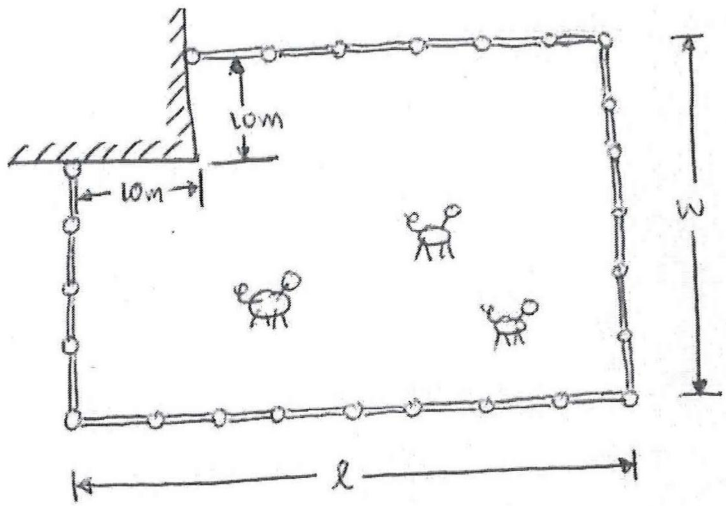
$$\boxed{T = 11.5}$$

sub backinto (1)

$$B = [11.5] + 21.5 = 33$$

$$\boxed{B = 33}$$

(5 marks) You are provided with 84.3 m of fencing and are required to build a fence as shown in the figure. Note, that a 10 m by 10 m corner does not need to be fenced. Further, the length should be 1.62 times larger than the width. What are the dimensions of the fence?



let:

ℓ = lenght as indicated in Figure

w = width as indicated in Figure.

so

$$84.3 = \ell + (\ell - 10) + w + (w - 10) \quad (1)$$

$$= 2\ell + 2w - 20$$

and

$$1.62w = \ell \quad (2)$$

sub (2) into (1):

$$84.3 = 2[1.62w] + 2w - 20$$

$$104.3 = 3.24w + 2w$$

$$5.24w = 104.3$$

$$\boxed{w = 19.9 \text{ m}}$$

sub back into (2)

$$\boxed{\ell = 1.62[19.9] = 32.2 \text{ m}}$$

so