Instructor:	Frank Secretain
Course:	Math 101
Assessment:	Final
Time allowed:	110 minutes
Devices allowed:	Pencil, pen, eraser, calculator
Notes from instructor:	Be neat. Show your work where needed. Box final answers.
Marks allocated:	7 questions worth 30 marks + 1 bonus question worth 1 mark
Percentage of final grade:	25% of final grade

Formula Sheet

Order of Operations

$$ac + bc = c(a + b)$$

exponents

$$a^n a^m = a^{n+m}$$

$$(a^n)^m = a^{nm}$$

$$(ab)^n = a^n b^n$$

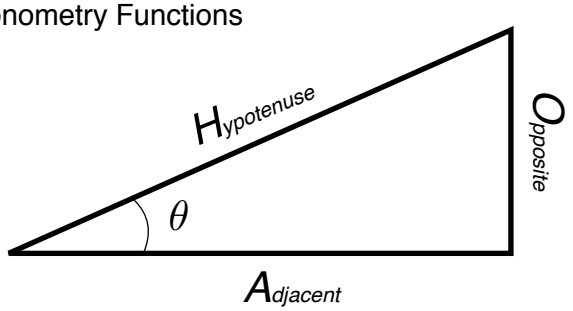
$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

radicals

$$a^{\frac{n}{m}} = \sqrt[m]{a^n}$$

Trigonometry Functions



$$sin(\theta) = \frac{O}{H} \qquad sin^{-1}\left(\frac{O}{H}\right) = \theta$$

$$cos(\theta) = \frac{A}{H} \qquad cos^{-1}\left(\frac{A}{H}\right) = \theta$$

$$tan(\theta) = \frac{O}{A} \qquad tan^{-1}\left(\frac{O}{A}\right) = \theta$$

Pythagoras Theorem

$$H^2 = O^2 + A^2$$

Relative Velocity

$$\vec{v}_{\frac{A}{C}} = \vec{v}_{\frac{A}{B}} + \vec{v}_{\frac{B}{C}}$$

Linear equations (Cramer's rule)

$$x_i = \frac{det(A_i)}{det(A)}$$

Forms of a 1st order polynomial

$$y = ax + b$$

Forms of a 2nd order polynomial

$$y = ax^2 + bx + c$$

$$y = a(x - h)^2 + k$$

$$y = (x - m)(x - n)$$

Unit Conversions

angles

$$2\pi = 6.28 \text{ rad} = 360^o$$

mass

$$1 \text{ kg} = 2.2 \text{ lbs.}$$

lengths

$$1 \text{ mile} = 1.6 \text{ km}$$

$$1 \text{ inch} = 2.54 \text{ cm}$$

$$1 \text{ m} = 3.3 \text{ ft}$$

volumes

$$1 \text{ gallon} = 3.78 \text{ Litres}$$

Frank Secretain's math course review

name: anonymous

Overall rating of this course

1	2	3	4	5	6	7	8	9	10

(worst course ever) (best course ever)

Things you liked about the course:

Things you didn't like about the course:

Other comments:

Thank you for your comments
Have a great holiday

(2 marks) Solve the each expression and keep the correct number of significant digits.

1.254 + (53.1)(0.010)

134.5 + (315.26) / (0.11)

(2 marks) Convert each number into scientific notation.

0.00120

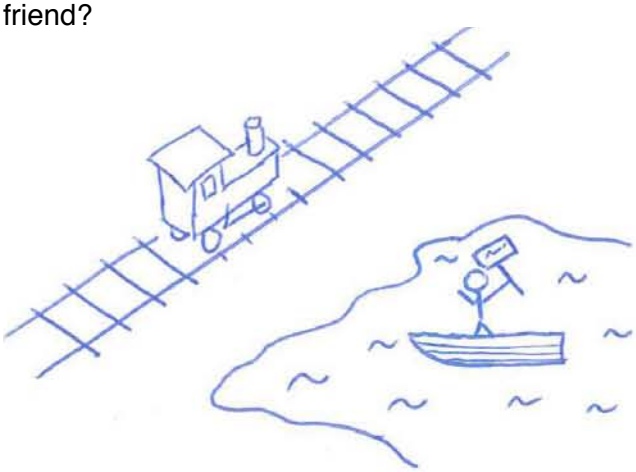
1200

(2 marks) Convert each of the numbers to the stated units.

$$\frac{\pi}{3} \rightarrow \text{degrees}$$

$$830\ \frac{\text{gal.}}{\text{cm}^3} \rightarrow \frac{\text{L}}{\text{mm}^3}$$

(5 marks) Your on a train travelling 20 m/s at 30 degrees North of East relative to the ground. Your friend is on a boat which is travelling 5 m/s West relative to the water. The water is travelling at 4 m/s at 10 degrees South of East relative to the ground. How fast are you travelling with respect to your friend?



(10 marks) Solve for x in the expressions:

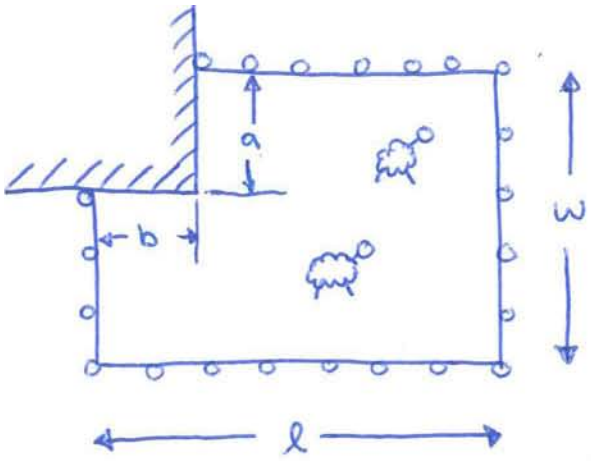
$$5x + 2(a^2 - a(a - b)) + \epsilon x = 7$$

$$y - a = 3 + \frac{2(a - x)}{a - b}\Lambda$$

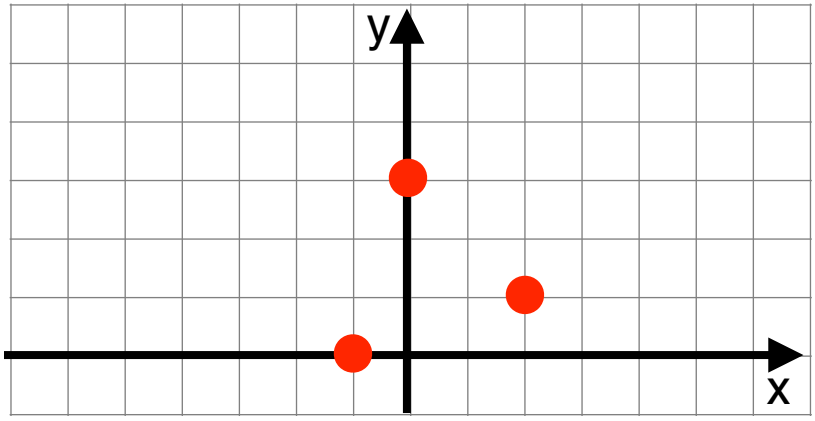
$$ma = R_x - \left[\frac{\frac{3}{2}Lm_1g + xm_2g}{L \sin\theta} \right] \cos\theta$$

$$\frac{1}{R_F} = \frac{1}{R_1 + x} + \frac{1}{R_2}$$

(5 marks) You are provided with 100 m of fencing and are required to build a fence as shown in the figure. The North West corner of the building does not need to be fenced as the wall is there. Further, the length should be 1.5 times larger than the width. What are the dimensions of the fence?



(4 marks) Find the equation of the parabola that passes through the points (–1, 0), (0, 3), and (2, 1)



(1 mark) BONUS:

what is the vertex of the parabola?

(2 marks) Solve the each expression and keep the correct number of significant digits.

$1.254 + \underset{3}{(53.1)}\underset{2}{(0.010)} = 1.254 + 0.531$

$= 1.785$

$\boxed{= 1.79}$

$134.5 + \underset{5}{(315.26)} / \underset{2}{(0.11)} = 134.5 + \frac{2866}{2}$

$= 3000.5$

$\boxed{= 3.0 \times 10^3}$

(2 marks) Convert each number into scientific notation.

0.00120

$\boxed{= 1.20 \times 10^{-3}}$

1200

$\boxed{= 1.2 \times 10^{3}}$

(2 marks) Convert each of the numbers to the stated units.

$\frac{\pi}{3} \rightarrow$ degrees

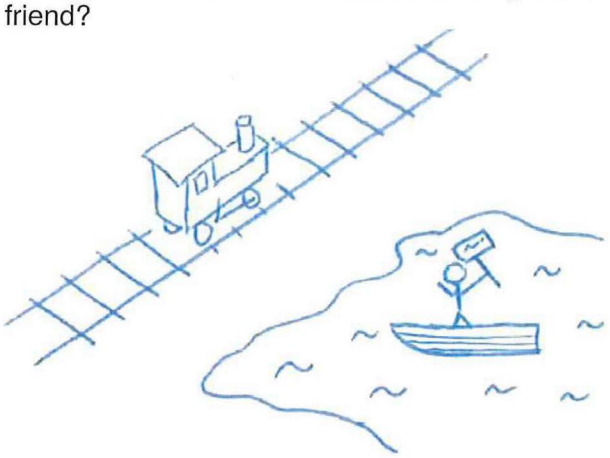
$\frac{\pi}{3}\left(\frac{360^\circ}{2\pi}\right) = 60^\circ$ $\boxed{60^\circ}$

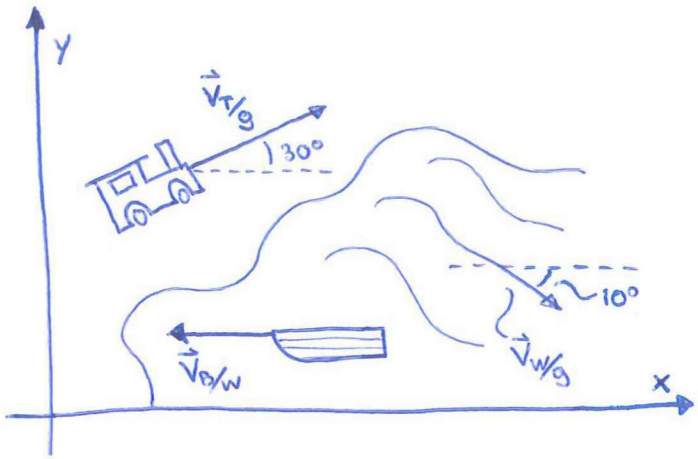
$830\ \frac{\text{gal.}}{\text{cm}^3} \rightarrow \frac{\text{L}}{\text{mm}^3}$

$830\ \frac{\text{gal.}}{\text{cm}^3}\left(\frac{3.78\ \text{L}}{1\ \text{gal.}}\right)\left(\frac{1\ \text{cm}}{10\ \text{mm}}\right)^3 = 3.14\ \frac{\text{L}}{\text{mm}^3}$

$\boxed{3.14\ \frac{\text{L}}{\text{mm}^3}}$

(5 marks) Your on a train travelling 20 m/s at 30 degrees North of East relative to the ground. Your friend is on a boat which is travelling 5 m/s West relative to the water. The water is travelling at 4 m/s at 10 degrees South of East relative to the ground. How fast are you travelling with respect to your friend?





Relative velocity:

$$\vec{V}_{T/B} = \vec{V}_{T/g} + \vec{V}_{g/w} + \vec{V}_{w/B}$$

or

$$\vec{V}_{B/T} = \vec{V}_{B/W} + \vec{V}_{W/g} + \vec{V}_{g/T}$$

so

$$\vec{V}_{T/B} = \vec{V}_{T/g} - \vec{V}_{W/g} - \vec{V}_{B/W}$$

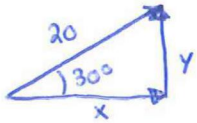
and

$$\vec{V}_{T/g} = 17.32\,\hat{x} + 10\,\hat{y}$$

$$-\vec{V}_{W/g} = -3.94\,\hat{x} + 0.69\,\hat{y}$$

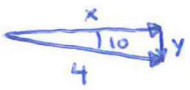
$$-\vec{V}_{B/W} = +5\,\hat{x} + 0\,\hat{y}$$

$$\vec{V}_{T/B} = 18.38\,\hat{x} + 10.69\,\hat{y}$$



$x = 20\cos(30) = 17.32$

$y = 20\sin(30) = 10$



$x = 4\cos(10) = 3.94$

$y = 4\sin(10) = 0.69$

$$\left|\vec{V}_{T/B}\right| = \sqrt{18.38^2 + 10.69^2}$$

$$= 21.3 \text{ m/s}$$

$$\angle V_{T/B} = \tan^{-1}\left(\frac{10.69}{18.38}\right)$$

$$= 30.2°$$

(10 marks) Solve for x in the expressions:

$$5x + 2(a^2 - a(a-b)) + \epsilon x = 7$$

$5x + 2(a^2 - a^2 + ab) + \epsilon x = 7$

$5x + \epsilon x = 7 - 2(ab)$

$x(5 + \epsilon) = 7 - 2ab$

$$x = \frac{7 - 2ab}{5 + \epsilon}$$

$$y - a = 3 + \frac{2(a-x)}{a-b}\Lambda$$

$y - a - 3 = \frac{2\Lambda(a-x)}{a-b}$

$2\Lambda(a-x) = (a-b)(y-a-3)$

$2\Lambda a - 2\Lambda x = (a-b)(y-a-3)$

$2\Lambda x = 2\Lambda a - (a-b)(y-a-3)$

$$x = a - \frac{(a-b)(y-a-3)}{2\Lambda}$$

$$ma = R_x - \left[\frac{\frac{3}{2}Lm_1g + xm_2g}{L\sin\theta}\right]\cos\theta$$

$$\left[\frac{\frac{3}{2}Lm_1g + xm_2g}{L\sin\theta}\right] = \frac{R_x - ma}{\cos\theta}$$

$$xm_2g = \left(\frac{R_x - ma}{\cos\theta}\right)L\sin\theta - \frac{3}{2}Lm_1g$$

$$x = \left(\frac{R_x - ma}{m_2g}\right)\left(\frac{L\sin\theta}{\cos\theta}\right) - \frac{3Lm_1}{2m_2}$$

$$= \frac{2LR_x\sin\theta - 2Lma\sin\theta - 3Lm_1g\cos\theta}{2m_2g\cos\theta}$$

$$\frac{1}{R_F} = \frac{1}{R_1 + x} + \frac{1}{R_2}$$

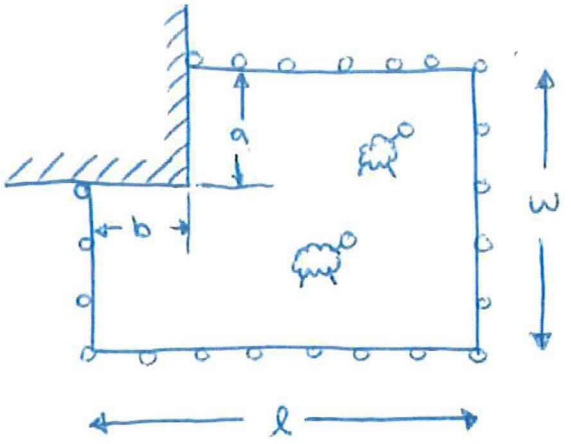
$$\frac{1}{R_1 + x} = \frac{1}{R_F} - \frac{1}{R_2}$$

$$R_1 + x = \left[\frac{1}{R_F} - \frac{1}{R_2}\right]^{-1} = \left[\frac{R_2 - R_F}{R_2R_F}\right]^{-1} = \frac{R_2R_F}{R_2 - R_F}$$

$$x = \frac{R_2R_F}{R_2 - R_F} - R_1$$

$$= \frac{R_2R_F + R_1R_F - R_1R_2}{R_2 - R_F}$$

(5 marks) You are provided with 100 m of fencing and are required to build a fence as shown in the figure. The North West corner of the building does not need to be fenced as the wall is there. Further, the length should be 1.5 times larger than the width. What are the dimensions of the fence?



$$2l + 2w - a - b = 100 \quad (1)$$

$$(1.5)(w) = l \quad (2)$$

sub. (2) into (1)

$$2[1.5w] + 2w - a - b = 100$$

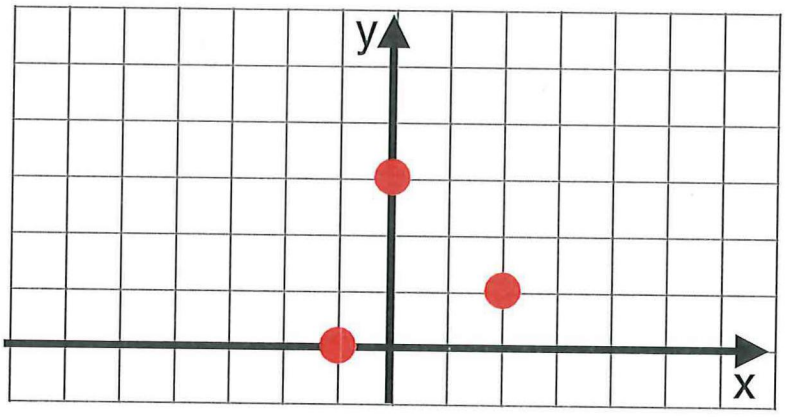
$$5w = 100 + a + b$$

$$\boxed{w = 20 + \frac{a+b}{5}}$$

sub back into (2)

$$\boxed{l = 30 + \frac{3(a+b)}{10}}$$

(4 marks) Find the equation of the parabola that passes through the points (–1, 0), (0, 3), and (2, 1)



equation of a parabola: $y = ax^2 + bx + c$

pt. $(-1,0)$: $0 = a - b + c$ (1)

pt. $(0,3)$: $3 = c$ (2) $\Rightarrow$ $\boxed{c = 3}$

pt. $(2,1)$: $1 = 4a + 2b + c$ (3)

sub. (2) into (1)

$0 = a - b + [3]$

$b = a + 3$ (1a)

sub (1a) & (2) into (3)

$1 = 4a + 2[a+3] + [3]$

$1 = 6a + 6 + 3$

$a = -\frac{8}{6} = -\frac{4}{3}$ (3a) $\Rightarrow$ $\boxed{a = -\frac{4}{3} \approx -1.33}$

back sub. into (1a)

$b = [-\frac{4}{3}] + 3 = \frac{5}{3}$ $\Rightarrow$ $\boxed{b = \frac{5}{3} \approx 1.67}$

so

$$\boxed{y = -\frac{4}{3}x^2 + \frac{5}{3}x + 3}$$

(1 mark) BONUS:

what is the vertex of the parabola?

$$y = -\frac{4}{3}x^2 + \frac{5}{3}x + 3 = a(x-h)^2 + k$$
$$= ax^2 - 2hax + ah^2 + k$$

x^2: $\quad -\frac{4}{3} = a \quad \Rightarrow \quad a = -\frac{4}{3} \cong 1.33$

x^1: $\quad \frac{5}{3} = -2ha \quad \Rightarrow \quad h = \frac{5}{8} = 0.625$

x^0: $\quad 3 = ah^2 + k \quad \Rightarrow \quad k = \frac{169}{48} \cong 3.52$

so

$$\boxed{y = -\frac{4}{3}\left(x - \frac{5}{8}\right)^2 + \frac{169}{48}}$$

and

$$\boxed{\text{vertex} = \left(\frac{5}{8}, \frac{169}{48}\right) \cong (0.63, 3.5)}$$