Instructor:	Frank Secretain
Course:	Math 101
Assessment:	Test 2
Time allowed:	110 minutes
Devices allowed:	Pencil, pen, eraser, calculator
Notes from instructor:	Be neat. Show your work where needed. Box final answers.
Marks allocated:	5 questions worth 30 marks
Percentage of final grade:	24% of final grade

Formula Sheet

Order of Operations

$$ac + bc = c(a + b)$$

exponents

$$a^n a^m = a^{n+m}$$

$$(a^n)^m = a^{nm}$$

$$(ab)^n = a^n b^n$$

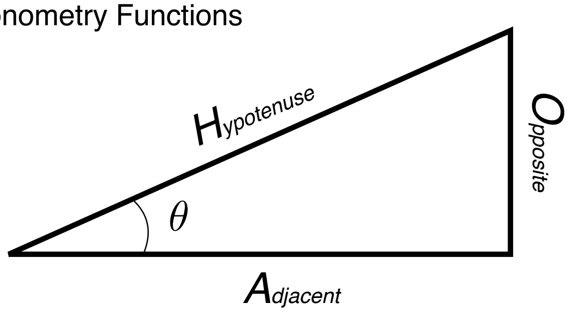
$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

radicals

$$a^{\frac{n}{m}} = \sqrt[m]{a^n}$$

Trigonometry Functions



$$sin(\theta) = \frac{O}{H} \qquad sin^{-1}\left(\frac{O}{H}\right) = \theta$$

$$cos(\theta) = \frac{A}{H} \qquad cos^{-1}\left(\frac{A}{H}\right) = \theta$$

$$tan(\theta) = \frac{O}{A} \qquad tan^{-1}\left(\frac{O}{A}\right) = \theta$$

Pythagoras Theorem

$$H^2 = O^2 + A^2$$

Relative Velocity

$$\vec{v}_{\frac{A}{C}} = \vec{v}_{\frac{A}{B}} + \vec{v}_{\frac{B}{C}}$$

Linear equations (Cramer's rule)

$$x_i = \frac{det(A_i)}{det(A)}$$

Forms of a 1st order polynomial

$$y = ax + b$$

Forms of a 2nd order polynomial

$$y = ax^2 + bx + c$$

$$y = a(x - h)^2 + k$$

$$y = (x - m)(x - n)$$

Unit Conversions

angles

$$2\pi = 6.28 \text{ rad} = 360^o$$

mass

$$1 \text{ kg} = 2.2 \text{ lbs.}$$

lengths

$$1 \text{ mile} = 1.6 \text{ km}$$

$$1 \text{ inch} = 2.54 \text{ cm}$$

$$1 \text{ m} = 3.3 \text{ ft}$$

volumes

$$1 \text{ gallon} = 3.78 \text{ Litres}$$

(12 marks) Solve for x in the expressions:

$$7x + 3x(x-2) - 3x^2 = 4$$

$$7x + \frac{3[x - 2(x-1)]}{2} = \frac{7x^2 - 7x}{x-1}$$

$$\frac{\alpha x - 1}{x - 1} + 1 = \eta + \epsilon$$

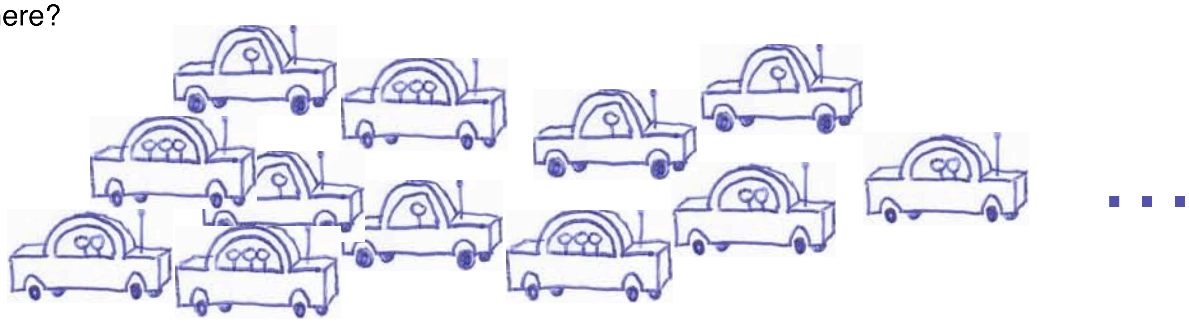
$$\phi x + \frac{2\gamma[4\Delta]^2(\mu - 1)}{4a + 1} = 4x + 2\alpha$$

(3 marks) Solve the system of linear equations:

$$3x - 4y = x - 3y - 1$$
$$7x + 10y - 50 = 2(x+y)$$

(5 marks) The combined total of characters (not including spaces) and words in this test is 1027. Further, there are 661 more characters than words. What is the word and character count in this test?

(5 marks) Suppose I took a picture of five hundred 3, 2 and 1 person cars (as illustrated below). The total number of people in the picture is 834 and the total number of tires is 2000. Lastly, the number of 2 person cars is the same as the number of 1 and 3 person cars combined. How many 1 person cars are there?



(5 marks) Simplify the following polynomial fraction using long division:

$$\frac{x^3 - 1}{x - 1} = ax^2 + bx + c + \frac{d}{x - 1}$$

(12 marks) Solve for x in the expressions:

$$7x + 3x(x-2) - 3x^2 = 4$$

$7x + \cancel{3x^2} - 6x - \cancel{3x^2} = 4$

$\boxed{x = 4}$

$$7x + \frac{3[x - 2(x-1)]}{2} = \frac{7x^2 - 7x}{x-1}$$

$7x + \frac{3(x - 2x + 2)}{2} = \frac{7x\cancel{(x-1)}}{\cancel{(x-1)}}$

$\cancel{14x} + 3(2-x) = \cancel{14x}$

$6 - 3x = 0$

$3x = 6$

$\boxed{x = 2}$

$$\frac{\alpha x - 1}{x - 1} + 1 = \eta + \epsilon$$

$$\frac{\alpha x - 1}{x - 1} = \eta + \epsilon - 1$$

$$\alpha x - 1 = (\eta + \epsilon - 1)(x - 1)$$

$$\alpha x - (\eta + \epsilon - 1)(x) = (\eta + \epsilon - 1)(-1) + 1$$

$$x(\alpha + 1 - \eta - \epsilon) = 1 - \eta - \epsilon + 1$$

$$\boxed{x = \frac{2 - \eta - \epsilon}{1 + \alpha - \eta - \epsilon}}$$

$$\phi x + \frac{2\gamma[4\Delta]^2(\mu - 1)}{4a + 1} = 4x + 2\alpha$$

$$\phi x - 4x = 2a - \frac{2\gamma[4\Delta]^2(\mu - 1)}{4a + 1}$$

$$x(\phi - 4) = 2\left(a - \frac{\gamma[4\Delta]^2(\mu - 1)}{4a + 1}\right)$$

$$\begin{aligned}
x &= \left(\frac{2}{\phi - 4}\right)\left(a - \frac{\gamma[4\Delta]^2(\mu - 1)}{4a + 1}\right) \\
&= \frac{2a}{\phi - 4} - \frac{2\gamma[4\Delta]^2(\mu - 1)}{(\phi - 4)(4a + 1)} \\
&= \frac{2a(4a + 1)}{(\phi - 4)(4a + 1)} - \frac{2\gamma[4\Delta]^2(\mu - 1)}{4\phi a + \phi - 16a - 4} \\
&= \frac{8a^2 + 2a - 2\gamma[4\Delta]^2(\mu - 1)}{4\phi a + \phi - 16a - 4}
\end{aligned}$$

(3 marks) Solve the system of linear equations:

$$3x - 4y = x - 3y - 1 \quad (1)$$
$$7x + 10y - 50 = 2(x+y) \quad (2)$$

Simplify (1)

$$3x - 4y = x - 3y - 1$$

$$2x - y = -1 \quad (1a)$$

simplify (2)

$$7x + 10y - 50 = 2x + 2y$$

$$5x + 8y = 50 \quad (2a)$$

solve for y in (1a)

$$y = 2x + 1 \quad (1b)$$

sub (1b) into (2a)

$$5x + 8[2x+1] = 50$$

$$5x + 16x + 8 = 50$$

$$21x = 42$$

$$\boxed{x = 2}$$

sub into (1b)

$$y = 2[2] + 1$$

$$\boxed{y = 5}$$

(5 marks) The combined total of characters (not including spaces) and words in this test is 1027. Further, there are 661 more characters than words. What is the word and character count in this test?

let

x = number of characters

y = number of words.

so

$$x + y = 1027 \quad (1)$$

$$x = y + 661 \quad (2)$$

sub (2) into (1)

$$[y + 661] + y = 1027$$

$$2y = 366$$

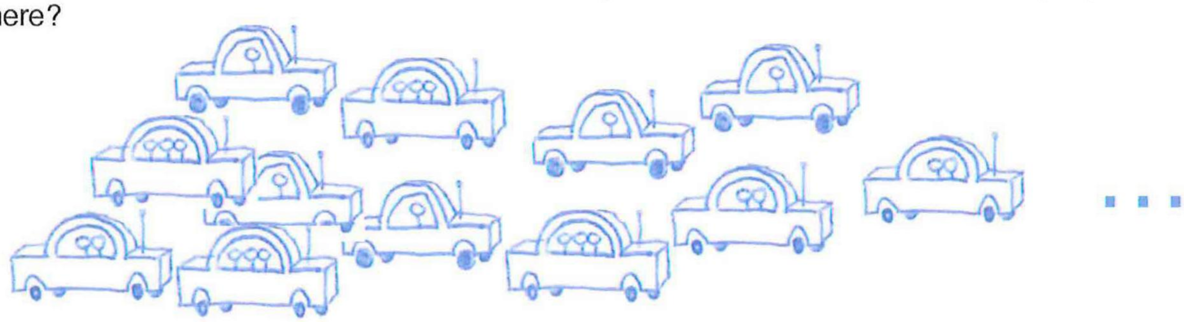
$$\boxed{y = 183}$$

sub into (2)

$$x = [183] + 661$$

$$\boxed{x = 844}$$

(5 marks) Suppose I took a picture of five hundred 3, 2 and 1 person cars (as illustrated below). The total number of people in the picture is 834 and the total number of tires is 2000. Lastly, the number of 2 person cars is the same as the number of 1 and 3 person cars combined. How many 1 person cars are there?



let

x = # of 1 person cars

y = # of 2 person cars

z = # of 3 person cars

so

$x + y + z = 500$ [cars] (1)

$1x + 2y + 3z = 834$ [people] (2)

$y = x + z$ [cars] (3)

sub (3) into (1) & (2)

$x + [x+z] + z = 500$

$2(x+z) = 500$

$x = 250 - z$ (1a)

$x + 2[x+z] + 3z = 834$

$3x + 5z = 834$ (2a)

sub (1a) into (2a)

$3[250 - z] + 5z = 834$

$750 + 2z = 834$

$\boxed{z = 42}$

sub into (1a) & (3)

$\boxed{x = 208}$

$\boxed{y = 250}$

$\boxed{\text{\# of 1 person cars} = 208}$

(5 marks) Simplify the following polynomial fraction using long division:

$$\frac{x^3-1}{x-1} = ax^2 + bx + c + \frac{d}{x-1}$$

$$(x-1)\left(\frac{x^3-1}{x-1}\right) = \left(ax^2 + bx + c + \frac{d}{x-1}\right)(x-1)$$

$$x^3-1 = ax^3 - ax^2 + bx^2 - bx + cx - c + d$$

x^3: $1 = a$ $\Rightarrow$ $a=1$

x^2: $0 = -a + b$ $\Rightarrow$ $b=1$

x^1: $0 = -b + c$ $\Rightarrow$ $c=1$

x^0: $-1 = -c + d$ $\Rightarrow$ $d=0$

so

$$\boxed{\frac{x^3-1}{x-1} = x^2 + x + 1}$$

(5 marks) Simplify the following polynomial fraction using long division:

$$\frac{x^3 - 1}{x - 1} = ax^2 + bx + c + \frac{d}{x - 1}$$

$$\begin{array}{r} x^2 + x + 1 \\ x-1 \overline{) x^3 + 0x^2 + 0x - 1} \\ \underline{x^3 - x^2} \\ x^2 + 0x \\ \underline{x^2 - x} \\ x - 1 \\ \underline{x - 1} \\ 0 \end{array}$$

so

$$\frac{x^3-1}{x-1} = x^2 + x + 1$$