Instructor: Frank Secretain
Course: Math 101
Assessment: Test 2
Time allowed: 110 minutes
Devices allowed: Pencil, pen, eraser, calculator

Marks allocated: 4 questions worth 25 marks
Percentage of final grade: 15% of final grade

Notes from instructor: Be neat. Show your work where needed. Box final answers.
Print your test and write answers in the space provided.
If you can't print, then use blank paper and copy the question number as it is written on the test and answer in the space provided as if the test was printed.

Questions: Give me a call on teams.

Submission: At the end of your test: scan or take pictures of your test pages in order. Compile email and send it to:

math101@franksecretain.ca
by 2:30 pm on December 4, 2020

Formula Sheet

Order of Operations

$$ac + bc = c(a + b)$$

exponents

$$a^n a^m = a^{n+m}$$

$$(a^n)^m = a^{nm}$$

$$(ab)^n = a^n b^n$$

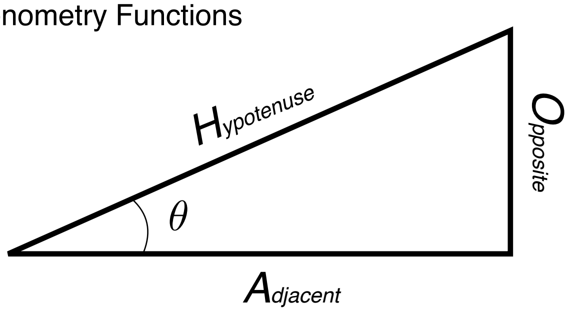
$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

radicals

$$a^{\frac{n}{m}} = \sqrt[m]{a^n}$$

Trigonometry Functions



$$sin(\theta) = \frac{O}{H} \qquad sin^{-1}\left(\frac{O}{H}\right) = \theta$$

$$cos(\theta) = \frac{A}{H} \qquad cos^{-1}\left(\frac{A}{H}\right) = \theta$$

$$tan(\theta) = \frac{O}{A} \qquad tan^{-1}\left(\frac{O}{A}\right) = \theta$$

Pythagoras Theorem

$$H^2 = O^2 + A^2$$

Relative Velocity

$$\vec{v}_{\frac{A}{C}} = \vec{v}_{\frac{A}{B}} + \vec{v}_{\frac{B}{C}}$$

Linear equations (Cramer's rule)

$$x_i = \frac{det(A_i)}{det(A)}$$

Forms of a 1st order polynomial

$$y = ax + b$$

Forms of a 2nd order polynomial

$$y = ax^2 + bx + c$$

$$y = a(x - h)^2 + k$$

$$y = (x - m)(x - n)$$

Unit Conversions

angles

$$2\pi = 6.28 \text{ rad} = 360^o$$

mass

$$1 \text{ kg} = 2.2 \text{ lbs.}$$

lengths

$$1 \text{ mile} = 1.6 \text{ km}$$

$$1 \text{ inch} = 2.54 \text{ cm}$$

$$1 \text{ m} = 3.3 \text{ ft}$$

volumes

$$1 \text{ gallon} = 3.78 \text{ Litres}$$

(12 marks) Solve for a in the expressions:

$$4a - 2(a-1) = 1$$

$$4b - b(a-2) = \beta$$

①

$$\frac{6(2 - a[b + 1])}{\alpha + 1} = b$$

$$\frac{2-a}{b+1}+2=\frac{7-a(2-\beta)}{\alpha+1}$$

$$y = \frac{a - a_o}{b - b_0} x + b$$

(3 marks) Solve the system of linear equations for x and y:

$$x(a+1)-y=2+a+y$$
$$7x + y(a-1) - 1 = 2(x+y)$$

(5 marks) Bill earns $12.50 an hour writing math tests and an additional $1.50 for every question that was written. Frank however, only earns $5.00 an hour for writing tests but gets an additional $4.00 per question written. How many questions needs to be written per hour so Bill and Frank make the same amount.

(5 marks) Simplify the following polynomial fraction using long division:

$$\frac{x^4}{2x-1}$$

⑦

(12 marks) Solve for a in the expressions:

$$4a - 2(a-1) = 1$$

$$4a - 2a + 2 = 1$$

$$2a = -1$$

$$\boxed{a = -\frac{1}{2}}$$

$$4b - b(a-2) = \beta$$

$$4b - ab + 2b = \beta$$

$$-ab = \beta - 6b$$

$$\boxed{a = \frac{6b - \beta}{b}}$$

①

$$\frac{6(2 - a[b+1])}{\alpha + 1} = b$$

$$6(2 - ab - a) = b(\alpha + 1)$$

$$12 - 6ab - 6a = b\alpha + b$$

$$-a\left[6b + 6\right] = b\alpha + b - 12$$

$$\boxed{a = \frac{12 - b\alpha - b}{6b + 6}}$$

②

$$\frac{2-a}{b+1}+2=\frac{7-a(2-\beta)}{\alpha+1}$$

$$(2-a)(\alpha+1) + 2(b+1)(\alpha+1) = [7 - a(2-\beta)](b+1)$$

$$2\alpha + 2 - a\alpha - a + 2(b+1)(\alpha+1) = 7(b+1) - a(2-\beta)(b+1)$$

$$-a\alpha - a + a(2-\beta)(b+1) = 7(b+1) - 2(b+1)(\alpha+1) - 2\alpha - 2$$

$$a\left[(2-\beta)(b+1) - \alpha - 1\right] = 7(b+1) - 2\left[(b+1)(\alpha+1) + \alpha + 1\right]$$

$$a = \frac{7(b+1) - 2\left[(b+1)(\alpha+1) + (\alpha+1)\right]}{(2-\beta)(b+1) - \alpha - 1}$$

$$\boxed{a = \frac{7(b+1) - 2(b+1)(\alpha+1) - 2(\alpha+1)}{(2-\beta)(b+1) - (\alpha+1)}}$$

$$y = \frac{a - a_o}{b - b_0}x + b$$

$$y(b-b_0) = (a-a_0)x + b(b-b_0)$$

$$y(b-b_0) = ax - a_0x + b(b-b_0)$$

$$ax = y(b-b_0) + a_0x - b(b-b_0)$$

$$\boxed{a = \frac{(y-b)(b-b_0) + a_0x}{x}}$$

④

(3 marks) Solve the system of linear equations for x and y:

$$x(a+1)-y=2+a+y \qquad (1)$$
$$7x + y(a-1) - 1 = 2(x+y) \qquad (2)$$

simplify (1):

$$(a+1)x - 2y = 2+a \qquad (1a)$$

simplify (2):

$$5x + (a-3)y = 1 \qquad (2a)$$

solve for x in (2a)

$$x = \frac{1}{5} - \frac{a-3}{5}y \qquad (2b)$$

sub (2b) into (1a)

$$(a+1)\left[\frac{1}{5} - \frac{a-3}{5}y\right] - 2y = 2+a$$

$$(a+1) - (a+1)(a-3)y - 10y = 10 + 5a$$

$$y\left((a+1)(a-3)+10\right) = a+1-10-5a$$

$$\boxed{y = \frac{-4a-9}{(a+1)(a-3)+10} = \frac{-4a-9}{a^2-2a+7}} \qquad (1b)$$

sub (1b) into (2b)

$$x = \frac{1}{5} - \frac{a-3}{5}\left[\frac{-4a-9}{a^2-2a+7}\right]$$

$$= \frac{a^2-2a+7 \quad -a+3}{5a^2-10a+35}$$

$$\boxed{x = \frac{a^2-3a+10}{5a^2-10a+35}}$$

(5 marks) Bill earns $12.50 an hour writing math tests and an additional $1.50 for every question that was written. Frank however, only earns $5.00 an hour for writing tests but gets an additional $4.00 per question written. How many questions needs to be written per hour so Bill and Frank make the same amount.

let:

h = hours worked

q = questions written

m = money earned

so

$m = (12^{\underline{50}})(h) + (1^{50})(q)$ bill

$m = (5^{\underline{00}})(h) + (4^{\underline{00}})(q)$ frank

and

$$(12^{\underline{50}})(h) + (1^{50})(q) = (5^{\underline{00}})(h) + (4^{\underline{00}})(q)$$

$$12^{\underline{50}} + (1^{50})\left(\frac{q}{h}\right) = 5^{\underline{00}} + (4^{\underline{00}})\left(\frac{q}{h}\right)$$

$$-2^{50}\frac{q}{h} = -7^{50}$$

$$\boxed{\frac{q}{h} = 3 \text{ questions/hour}}$$

(5 marks) Simplify the following polynomial fraction using long division:

$$(2x-1)\left(\frac{x^4}{2x-1}\right) = \left(ax^3 + bx^2 + cx + d + \frac{e}{2x-1}\right)(2x-1)$$

$$x^4 = 2ax^4 - ax^3 + 2bx^3 - bx^2 + 2cx^2 - cx + 2dx - d + e$$

x^4: $1 = 2a \Rightarrow a = \frac{1}{2}$

x^3: $0 = -a + 2b \Rightarrow b = \frac{1}{4}$

x^2: $0 = -b + 2c \Rightarrow c = \frac{1}{8}$

x^1: $0 = -c + 2d \Rightarrow d = \frac{1}{16}$

x^0: $0 = -d + e \Rightarrow e = \frac{1}{16}$

so

$$\boxed{\frac{x^4}{2x-1} = \frac{1}{2}x^3 + \frac{1}{4}x^2 + \frac{1}{8}x + \frac{1}{16} + \frac{1/16}{2x-1}}$$

(5 marks) Simplify the following polynomial fraction using long division:

$$\frac{x^4}{2x-1}$$

$$
\begin{array}{rl}
 & \frac{1}{2}x^3 + \frac{1}{4}x^2 + \frac{1}{8}x + \frac{1}{16} \\
\frac{x^4}{2x-1} \; = \; 2x-1 \,\big) & \overline{x^4 + 0x^3 + 0x^2 + 0x + 0} \\
 & -\left(x^4 - \frac{1}{2}x^3\right) \downarrow \\
 & \underline{\hphantom{-\left(x^4 - \frac{1}{2}x^3\right)}} \\
 & 0 \quad \frac{1}{2}x^3 + 0x^2 \\
 & \quad -\left(\frac{1}{2}x^3 - \frac{1}{4}x^2\right) \downarrow \\
 & \quad \underline{\hphantom{-\left(\frac{1}{2}x^3 - \frac{1}{4}x^2\right)}} \\
 & \quad\quad 0 \quad \frac{1}{4}x^2 + 0x \\
 & \quad\quad -\left(\frac{1}{4}x^2 - \frac{1}{8}x\right) \downarrow \\
 & \quad\quad \underline{\hphantom{-\left(\frac{1}{4}x^2 - \frac{1}{8}x\right)}} \\
 & \quad\quad\quad 0 \quad \frac{1}{8}x + 0 \\
 & \quad\quad\quad -\left(\frac{1}{8}x - \frac{1}{16}\right) \\
 & \quad\quad\quad \underline{\hphantom{-\left(\frac{1}{8}x - \frac{1}{16}\right)}} \\
 & \quad\quad\quad\quad 0 \quad \frac{1}{16}
\end{array}
$$

so

$$\boxed{\frac{x^4}{2x-1} = \frac{1}{2}x^3 + \frac{1}{4}x^2 + \frac{1}{8}x + \frac{1}{16} + \frac{1/16}{2x-1}}$$