Instructor:	Frank Secretain
Course:	Math 101
Assessment:	Final Test
Time allowed:	110 minutes
Devices allowed:	Pencil, pen, eraser, calculator
Notes from instructor:	Be neat. Show your work where needed. Box final answers.
Marks allocated:	7 questions worth 25 marks + 1 bonus worth 2 marks.
Percentage of final grade:	25% of final grade

Formula Sheet

Order of Operations

$$ac + bc = c(a+b)$$

exponents

$$a^n a^m = a^{n+m}$$

$$(a^n)^m = a^{nm}$$

$$(ab)^n = a^n b^n$$

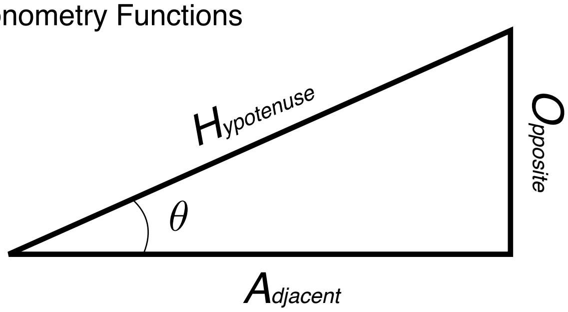
$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

radicals

$$a^{\frac{n}{m}} = \sqrt[m]{a^n}$$

Trigonometry Functions



$$sin(\theta) = \frac{O}{H} \qquad sin^{-1}\left(\frac{O}{H}\right) = \theta$$

$$cos(\theta) = \frac{A}{H} \qquad cos^{-1}\left(\frac{A}{H}\right) = \theta$$

$$tan(\theta) = \frac{O}{A} \qquad tan^{-1}\left(\frac{O}{A}\right) = \theta$$

Pythagoras Theorem

$$H^2 = O^2 + A^2$$

Relative Velocity

$$\vec{v}_{\frac{A}{C}} = \vec{v}_{\frac{A}{B}} + \vec{v}_{\frac{B}{C}}$$

Linear equations (Cramer's rule)

$$x_i = \frac{det(A_i)}{det(A)}$$

Forms of a 1st order polynomial

$$y = ax + b$$

Forms of a 2nd order polynomial

$$y = ax^2 + bx + c$$

$$y = a(x-h)^2 + k$$

$$y = (x-m)(x-n)$$

Unit Conversions

angles

$$2\pi = 6.28 \text{ rad} = 360^o$$

mass

$$1 \text{ kg} = 2.2 \text{ lbs.}$$

lengths

$$1 \text{ mile} = 1.6 \text{ km}$$

$$1 \text{ inch} = 2.54 \text{ cm}$$

$$1 \text{ m} = 3.3 \text{ ft}$$

volumes

$$1 \text{ gallon} = 3.78 \text{ Litres}$$

(3 marks) Match the "type of number" with the best "example number". Draw a line to match the "type of number" to the "example number" to indicate your answer.

rational	2.2
real	2
natural	$\sqrt{2}$

(2 marks) Solve the each expression and keep the correct number of significant digits.

(34)(1.45) =

14.0 + (60.1)(0.1) =

let:

$$15.6\tau = \Lambda \qquad 4.6\gamma = 3.1\beta$$

$$0.087\epsilon = 2.3\Lambda \qquad 3.1\theta = 2.1\Phi$$

(2 marks) Convert each of the numbers to the stated units.

$$1.3\frac{\beta}{second} \rightarrow \frac{\gamma}{hour}$$

$$3.2\frac{m^2}{\tau} \rightarrow \frac{ft^2}{\epsilon}$$

(2 marks) Re-write the equation in computer syntax with the minimum number of characters, Do not simplify or rearrange the equation.

$$\frac{a-1}{b^2+1}-c=1$$

$$a+\frac{b^{b+c}+1}{a}=b$$

(8 marks) Solve for x in the expressions:

$$y=\frac{ax}{\Delta}+b$$

$$y = a\frac{x-1}{\Delta - 2} + xb^2$$

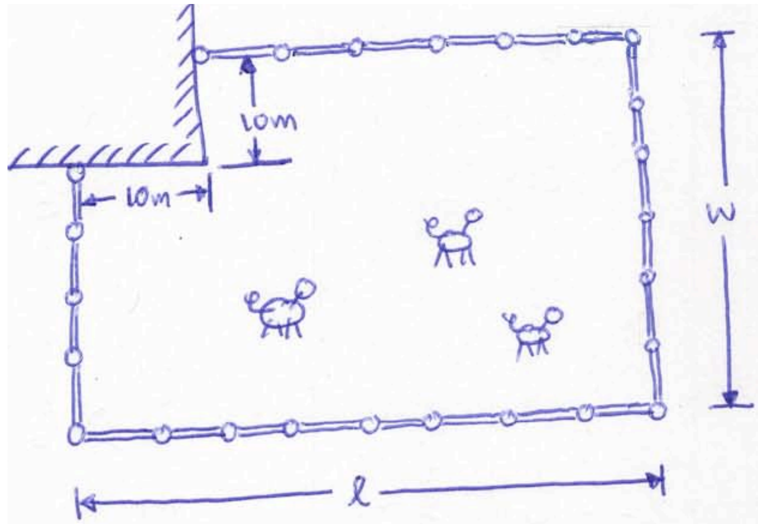
$$\big(a + \sin(2x)\big)^2 + b^2 = c^2$$

$$1 - b^2 - \frac{a(x-1)^2 + cb^2}{c-1} - 2 = 1 + a$$

(3 marks) Solve the system of linear equations for a, b and c:

$$
\begin{aligned}
a + 2b + 2c &= 3 \\
b - c. &= -5 \\
a. + 2b \quad &= -3
\end{aligned}
$$

(5 marks) You are provided with 84.3 m of fencing and are required to build a fence as shown in the figure. Note, that a 10 m by 10 m corner does not need to be fenced. Further, the length should be 1.62 times larger than the width. What are the dimensions of the fence?



BONUS:

(2 marks) Carson walked 50 m at 30^0 North of West at 2 m/s and then ran in the direction of 40^0 East of North at 6 m/s. Frank first walked 40 m East at 3 m/s and then ran in the direction of 30^0 West of North at 7 m/s. How long should Carson initially wait so that their paths cross at the same time (i.e. they meet at the exact same spot when they are both running)?

(3 marks) Match the "type of number" with the best "example number". Draw a line to match the "type of number" to the "example number" to indicate your answer.

rational	2.2
real	2
natural	$\sqrt{2}$

(2 marks) Solve the each expression and keep the correct number of significant digits.

$\underbrace{(34)}_{2}\underbrace{(1.45)}_{3} = \underbrace{49.3}_{2}$

$\boxed{= 49}$

$14.0 + \underbrace{(60.1)}_{3}\underbrace{(0.1)}_{1} = 14.0_{+1} + 6.01_{\ \mathsf{T}\ 0_0}$

$= 20.01_{0_0}$

$\boxed{= 2.0 \times 10^1}$

let:

$$15.6\tau = \Lambda \qquad 4.6\gamma = 3.1\beta$$

$$0.087\epsilon = 2.3\Lambda \qquad 3.1\theta = 2.1\Phi$$

(2 marks) Convert each of the numbers to the stated units.

$$1.3\frac{\beta}{second} \rightarrow \frac{\gamma}{hour}$$

$$1.3\,\frac{\cancel{\beta}}{\cancel{s}}\left(\frac{4.6\,\gamma}{3.1\,\cancel{\beta}}\right)\left(\frac{60\,\cancel{s}}{1\,\cancel{m}}\right)\left(\frac{60\,\cancel{m}}{1\,h}\right) = 6944.51$$

$$\boxed{= 6940}$$

$$3.2\frac{m^2}{\tau} \rightarrow \frac{ft^2}{\epsilon}$$

$$3.2\,\frac{\cancel{m^2}}{\cancel{\tau}}\left(\frac{3.3^2\,ft^2}{1^2\,\cancel{m^2}}\right)\left(\frac{15.6\,\cancel{\tau}}{1\,\cancel{\Lambda}}\right)\left(\frac{2.3\,\cancel{\Lambda}}{0.087\,\epsilon}\right) = 14371.7959$$

$$\boxed{= 14400}$$

(2 marks) Re-write the equation in computer syntax with the minimum number of characters, Do not simplify or rearrange the equation.

$$\frac{a-1}{b^2+1} - c = 1$$

$$\boxed{(a-1)/(b^2+1) - c = 1}$$

$$a + \frac{b^{b+c}+1}{a} = b$$

$$\boxed{a + (b\char`^(b+c)+1)/a = b}$$

(8 marks) Solve for x in the expressions:

$$y = \frac{ax}{\Delta} + b$$

$$\frac{ax}{\Delta} = y - b$$

$$\boxed{x = \frac{\Delta}{a}(y-b)}$$

$$y = a\frac{x-1}{\Delta - 2} + xb^2$$

$$y(\Delta-2) = a(x-1) + xb^2(\Delta-2)$$

$$ax - a + xb^2(\Delta-2) = y(\Delta-2)$$

$$x\left(a + b^2(\Delta-2)\right) = y(\Delta-2) + a$$

$$\boxed{x = \frac{y(\Delta-2)+a}{b^2(\Delta-2)+a}}$$

$$\left(a + \sin(2x)\right)^2 + b^2 = c^2$$

$$\left(a + \sin(2x)\right)^2 = c^2 - b^2$$

$$a + \sin(2x) = \sqrt{c^2-b^2}$$

$$\sin(2x) = \sqrt{c^2-b^2} - a$$

$$2x = \sin^{-1}\left(\sqrt{c^2-b^2} - a\right)$$

$$\boxed{x = \frac{1}{2}\sin^{-1}\left(\sqrt{c^2-b^2} - a\right)}$$

$$1 - b^2 - \frac{a(x-1)^2 + cb^2}{c-1} - 2 = 1 + a$$

$$-\frac{a(x-1)^2 + cb^2}{c-1} = 2 + a + b^2$$

$$-\left(a(x-1)^2 + cb^2\right) = (2 + a + b^2)(c-1)$$

$$-a(x-1)^2 - cb^2 = (2 + a + b^2)(c-1)$$

$$-a(x-1)^2 = (2 + a + b^2)(c-1) + cb^2$$

$$(x-1)^2 = -\frac{1}{a}\left((2 + a + b^2)(c-1) + cb^2\right)$$

$$x - 1 = \sqrt{-\frac{1}{a}\left[(2 + a + b^2)(c-1) + cb^2\right]}$$

$$\boxed{\begin{aligned} x &= \sqrt{-\frac{1}{a}\left[(2+a+b^2)(c-1) + cb^2\right]} + 1 \\ &= \sqrt{-\frac{1}{a}\left[2c - 2 + ac - a - b^2 + 2cb^2\right]} + 1 \end{aligned}}$$

(3 marks) Solve the system of linear equations for a, b and c:

$$a + 2b + 2c = 3 \quad (1)$$
$$b - c = -5 \quad (2)$$
$$a + 2b = -3 \quad (3)$$

solve for a in (3)

$$a = -3 - 2b \quad (3a)$$

sub (3a) into (1)

$$[-3 - 2b] + 2b + 2c = 3$$

$$-3 \cancel{-2b} + \cancel{2b} + 2c = 3$$

$$2c = 6$$

$$\boxed{c = 3} \quad (1a)$$

sub (1a) into (2)

$$b - [3] = -5$$

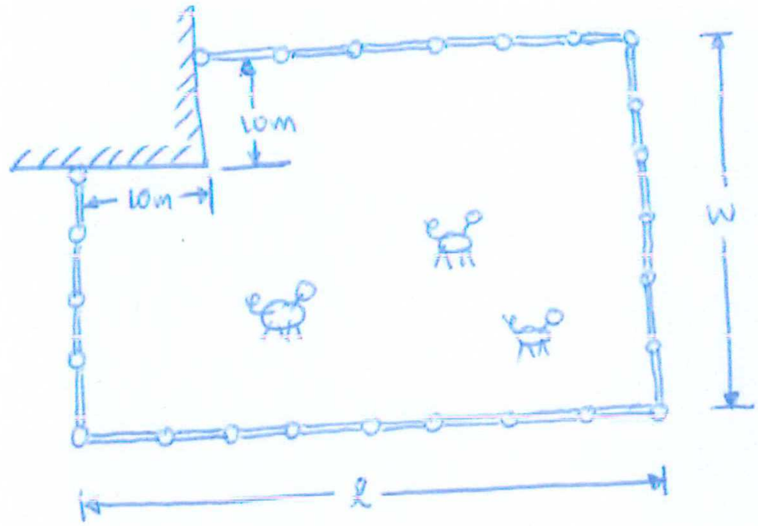
$$\boxed{b = -2} \quad (2a)$$

sub (2a) into (3a)

$$a = -3 - 2[-2]$$

$$\boxed{a = 1} \quad (3b)$$

(5 marks) You are provided with 84.3 m of fencing and are required to build a fence as shown in the figure. Note, that a 10 m by 10 m corner does not need to be fenced. Further, the length should be 1.62 times larger than the width. What are the dimensions of the fence?



let

l = lenght of fence

w = width of fence

So

$$l = (1.62)(w) \qquad (1)$$

$$p = l + (l-10) + w + (w-10) = \text{primeter} = 84.3 \qquad (2)$$

sub (1) into (2)

$$84.3 = [1.62w] + [1.62w] - 10 + w + w - 10$$

$$84.3 = 5.24w - 20$$

$$5.24w = 104.3$$

$$\boxed{w = 19.9 \text{ m}} \qquad (2a)$$

sub (2a) into (1)

$$\boxed{l = 32.2 \text{ m}} \qquad (1a)$$

t		32			
dt		-10			
	carson		frank		
t	x	y	x	y	
	32	-16.303779	57.1745331	9.6	52.5389122
	33	-12.447053	61.7707997	6.1	58.60109
	34	-8.5903276	66.3670664	2.6	64.6632678
	35	-4.733602	70.9633331	-0.9	70.7254457
	36	-0.8768763	75.5595997	-4.4	76.7876235
	37	2.97984935	80.1558664	-7.9	82.8498013