Instructor:	Frank Secretain
Course:	math 101
Assessment:	Test 2
Time allowed:	110 minutes
Devices allowed:	Pencil, pen, eraser, calculator
Notes from instructor:	Be neat. Show your work where needed. Box final answers.
Marks allocated:	5 questions worth 25 marks
Percentage of final grade:	25% of final grade

Formula Sheet

Order of Operations

$$ac + bc = c(a + b)$$

exponents

$$a^n a^m = a^{n+m}$$

$$(a^n)^m = a^{nm}$$

$$(ab)^n = a^n b^n$$

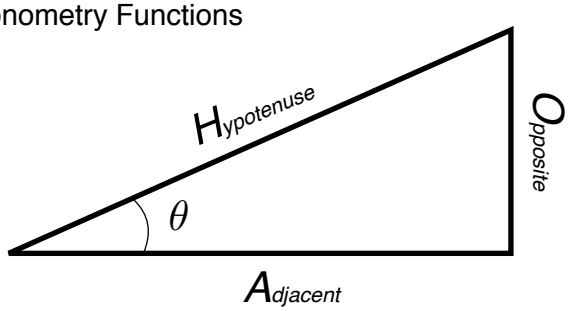
$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

radicals

$$a^{\frac{n}{m}} = \sqrt[m]{a^n}$$

Trigonometry Functions



$$sin(\theta) = \frac{O}{H} \qquad sin^{-1}\left(\frac{O}{H}\right) = \theta$$

$$cos(\theta) = \frac{A}{H} \qquad cos^{-1}\left(\frac{A}{H}\right) = \theta$$

$$tan(\theta) = \frac{O}{A} \qquad tan^{-1}\left(\frac{O}{A}\right) = \theta$$

Pythagoras Theorem

$$H^2 = O^2 + A^2$$

Relative Velocity

$$\vec{v}_{\frac{A}{C}} = \vec{v}_{\frac{A}{B}} + \vec{v}_{\frac{B}{C}}$$

Linear equations (Cramer's rule)

$$x_i = \frac{det(A_i)}{det(A)}$$

Forms of a 1st order polynomial

$$y = ax + b$$

Forms of a 2nd order polynomial

$$y = ax^2 + bx + c$$

$$y = a(x - h)^2 + k$$

$$y = (x - m)(x - n)$$

Unit Conversions

angles

$$2\pi = 6.28 \text{ rad} = 360^o$$

mass

$$1 \text{ kg} = 2.2 \text{ lbs.}$$

lengths

$$1 \text{ mile} = 1.6 \text{ km}$$

$$1 \text{ inch} = 2.54 \text{ cm}$$

$$1 \text{ m} = 3.3 \text{ ft}$$

volumes

$$1 \text{ gallon} = 3.78 \text{ Litres}$$

(8 marks) Solve for x in the following equations:

$$\frac{a(x-2)^2}{(b-2)^2} + \cos(\theta) - \Gamma = 2$$

$$a\frac{x-x_o}{y-y_o} + b = 2x$$

$$\frac{\gamma x - \beta}{x - 2} + 3\alpha = \eta + \epsilon$$

$$a = \frac{3(a - x(b + 2))}{x - 1} + 1$$

(5 marks) Solve the system of linear equations for x and y:

$$3x + 2y = 1$$
$$2x - y = 2$$

(5 marks) Solve the system of linear equations for x and y:

$$1 + \frac{2(x + 3(y - 3)) + 2}{y - 3} = 2$$

$$3(x + 1) - 1 = 2y$$

(5 marks) A woman owns 38 pets. Each of her pets is either a cat or a bird. If the pets have a total of 118 legs, how many cats and how many birds does the woman own?

(5 marks) A store sells two different types of coffee beans; the more expensive one sells for $8 per pound, and the cheaper one sells for $4 per pound. The beans are mixed to provide a mixture that sells for $6.40 per pound. What is the ratio of the expensive to cheap coffee beans?

(8 marks) Solve for x in the following equations:

$$\frac{a(x-2)^2}{(b-2)^2} + \cos(\theta) - \Gamma = 2$$

$$\frac{a(x-2)^2}{(b-2)^2} = 2 + \Gamma - \cos(\theta)$$

$$(x-2)^2 = \frac{(2+\Gamma-\cos(\theta))(b-2)^2}{a}$$

$$\boxed{x = \sqrt{\frac{(2+\Gamma-\cos(\theta))(b-2)^2}{a}} + 2}$$

$$a\frac{x-x_o}{y-y_o} + b = 2x$$

$$a(x-x_0) + b(y-y_0) = 2x(y-y_0)$$

$$ax - ax_0 + b(y-y_0) = 2x(y-y_0)$$

$$ax - 2x(y-y_0) = ax_0 - b(y-y_0)$$

$$x(a - 2(y-y_0)) = ax_0 - b(y-y_0)$$

$$\boxed{x = \frac{ax_0 - b(y-y_0)}{a - 2(y-y_0)}}$$

$$\frac{\gamma x - \beta}{x - 2} + 3\alpha = \eta + \epsilon$$

$$\gamma x - \beta + 3\alpha(x-2) = (\eta + \epsilon)(x-2)$$

$$\gamma x - \beta + 3\alpha x - 6\alpha = x(\eta + \epsilon) - 2(\eta + \epsilon)$$

$$\gamma x + 3\alpha x - x(\eta + \epsilon) = \beta + 6\alpha - 2(\eta + \epsilon)$$

$$x\left(\gamma + 3\alpha - (\eta + \epsilon)\right) = \beta + 6\alpha - 2(\eta + \epsilon)$$

$$\boxed{x = \frac{\beta + 6\alpha - 2(\eta + \epsilon)}{\gamma + 3\alpha - (\eta + \epsilon)}}$$

$$a = \frac{3(a - x(b+2))}{x - 1} + 1$$

$$(x-1)a = 3(a - x(b+2)) + (x-1)$$

$$ax - a = 3a - 3x(b+2) + x - 1$$

$$ax - a = 3a - 3xb - 6x + x - 1$$

$$ax + 3bx + 5x = a + 3a - 1$$

$$x(a + 3b + 5) = 4a - 1$$

$$\boxed{x = \frac{4a - 1}{a + 3b + 5}}$$

(2 marks) Solve the system of linear equations for x and y:

$$3x + 2y = 1 \quad (1)$$
$$2x - y = 2 \quad (2)$$

solve for y in (2)

$$2x = 2 + y$$

$$y = 2x - 2 \quad (2a)$$

sub (2a) into (1)

$$3x + 2[2x - 2] = 1$$

$$3x + 4x - 4 = 1$$

$$7x = 5$$

$$\boxed{x = \frac{5}{7} = 0.7143} \quad (1a)$$

sub (1a) into (2a)

$$y = 2\left[\frac{5}{7}\right] - 2$$

$$= \frac{10}{7} - 2$$

$$= \frac{10}{7} - \frac{14}{7}$$

$$\boxed{y = \frac{-4}{7} = -0.5714} \quad (2b)$$

(5 marks) Solve the system of linear equations for x and y:

$$1 + \frac{2(x + 3(y-3)) + 2}{y-3} = 2 \qquad (1)$$

$$3(x+1) - 1 = 2y \qquad (2)$$

expand (1)

$$2(x + 3(y-3)) + 2 = y - 3$$

$$2x + 6(y-3) + 2 = y - 3$$

$$2x + 6y - 18 + 2 = y - 3$$

$$2x + 5y = 13 \qquad (1a)$$

expand (2)

$$3x + 3 - 1 = 2y$$

$$3x - 2y = -2 \qquad (2a)$$

solve for y in (2a)

$$y = \frac{3x + 2}{2} \qquad (2b)$$

sub (2b) into (1a)

$$2x + 5\left[\frac{3x+2}{2}\right] = 13$$

$$4x + 5(3x+2) = 26$$

$$4x + 15x + 10 = 26$$

$$19x = 16$$

$$x = \frac{16}{19} = 0.8421 \qquad (1b)$$

sub (1b) into (2a)

$$3\left[\frac{16}{19}\right] - 2y = -2$$

$$-2y = -\frac{86}{19}$$

$$y = \frac{43}{19} = 2.263 \qquad (2c)$$

(5 marks) A woman owns 38 pets. Each of her pets is either a cat or a bird. If the pets have a total of 118 legs, how many cats and how many birds does the woman own?

let:

c = # of cats she owns.

b = # of birds she owns.

so:

$$c + b = 38 \qquad (1)$$

$$4c + 2b = 118 \qquad (2)$$

solve for c in (1):

$$c = 38 - b \qquad (1a)$$

sub (1a) into (2):

$$4[38 - b] + 2b = 118$$

$$152 - 4b + 2b = 118$$

$$-2b = -34$$

$$\boxed{b = 17} \qquad (2a)$$

sub (2a) into (1a):

$$c = 38 - [17]$$

$$\boxed{c = 21} \qquad (1b)$$

(5 marks) A store sells two different types of coffee beans; the more expensive one sells for $8 per pound, and the cheaper one sells for $4 per pound. The beans are mixed to provide a mixture that sells for $6.40 per pound. What is the ratio of the expensive to cheap coffee beans?

let: x = pounds of expensive beans.

y = pounds of cheap beans.

so

$$8x + 4y = 6.40\,t$$

(where t is the total amount of beans)

$$x + y = t$$

let:

t = total amount bought = 1 ← can be anything.

so:

$$8x + 4y = 6.40 \quad (1)$$
$$x + y = 1 \quad (2)$$

sub (2) into (1)

$$8[1 - y] + 4y = 6.40$$
$$8 - 8y + 4y = 6.40$$
$$-4y = -1.6$$
$$\boxed{y = 0.4} \quad (1a)$$
$$\boxed{x = 0.6}$$

so:

$$\text{ratio} = \frac{x}{y} = \frac{0.6}{0.4} = \frac{3}{2} = 1.5 = 150\,\%$$