Instructor:	Frank Secretain
Course:	Math 20
Assessment:	Test 2
Time allowed:	110 minutes
Devices allowed:	Pencil, pen, eraser, calculator
Marks allocated:	5 questions worth 20 marks
Percentage of final grade:	15% of final grade
Notes from instructor:	Be neat. Show your work where needed. Box final answers. Print your test and write answers in the space provided. If you can't print, then use blank paper and copy the question number as it is written on the test and answer in the space provided as if the test was printed.
Questions:	Give me a call on teams.
Submission:	At the end of your test: scan or take pictures of your test pages in order. Compile email and send it to: math20@franksecretain.ca by 10:30 am on December 4, 2020

Formula Sheet

Arithmetic Series

$$a_n = a_1 + (n-1)k$$
$$S_n = \sum_{i=1}^{n} a_1 + (i-1)k$$
$$= \frac{n}{2}(a_1 + a_n)$$

Geometric Series

$$a_n = a_1 r^{n-1}$$
$$S_n = \sum_{i=1}^{n} a_1 r^{i-1}$$
$$= a_1 \frac{1-r^n}{1-r}$$

Binomial Theorem

$$(x+y)^n = \sum_{k=0}^{n} \frac{n!}{(n-k)!k!} x^{n-k} y^k$$

Line equation

$$y = ax + b$$

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Definition of the derivative

$$\frac{d}{dx} f(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Rules of differentiation

$$\frac{d}{dx}(f(x)g(x)) = f(x)\frac{d}{dx}(g(x)) + g(x)\frac{d}{dx}(f(x))$$ (product rule)

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}(f(x)) - f(x)\frac{d}{dx}(g(x))}{(g(x))^2}$$ (quotient rule)

$$\frac{d}{dx}(f(g(x))) = \frac{d}{dx}(f(g(x)))\frac{d}{dx}(g(x))$$ (chain rule)

Derivatives of select functions

$$\frac{d}{dx}(ax^n) = anx^{n-1}$$
$$\frac{d}{dx}(sin(x)) = cos(x)$$
$$\frac{d}{dx}(cos(x)) = -sin(x)$$
$$\frac{d}{dx}(tan(x)) = \frac{1}{(cos(x))^2}$$
$$\frac{d}{dx}(a^x) = a^x ln(a)$$
$$\frac{d}{dx}(log_a x) = \frac{1}{xln(a)}$$

Integrals of select functions

$$\int ax^n dx = \begin{cases} \frac{a}{n+1}x^{n+1} & , n \neq -1 \\ aln(|x|) & , n = -1 \end{cases}$$ (polynomials)

$$\int sin(ax)dx = -\frac{1}{a}cos(ax)$$
$$\int cos(ax)dx = \frac{1}{a}sin(ax)$$ (trigonometry)
$$\int tan(ax)dx = \frac{1}{a}ln(|sec(ax)|)$$

$$\int a^x dx = \frac{1}{ln(a)}a^x$$ (exponentials)
$$\int ln(x)dx = xln(x) - x$$

Taylor series expansion

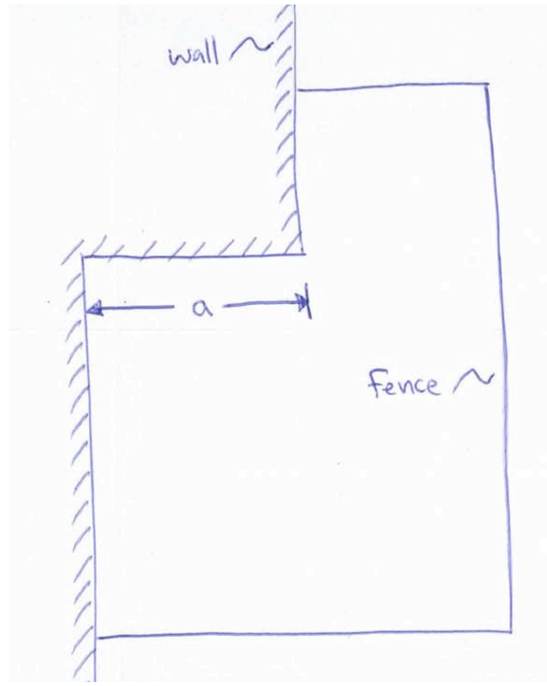
$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_o)}{n!}(x - x_o)^n$$

Integration by parts

$$\int udv = uv - \int vdu$$

(5 marks) Determine the dimensions of the fence to maximize the area given “L” meters of fencing. Parameters “a” and “L” should be set to:

question 1: a=0m, L=20m
question 2: a=2m, L=40m



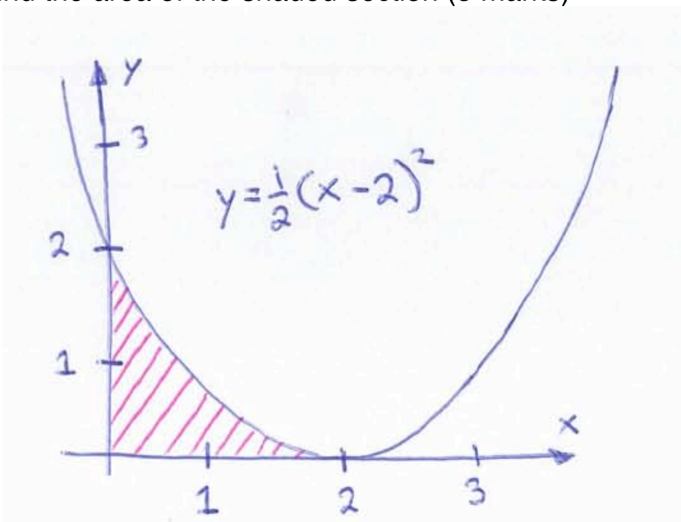
Integrate (2 marks)

$$\int \frac{1}{2}x^2 + sin(2x) - \beta x \; dx$$

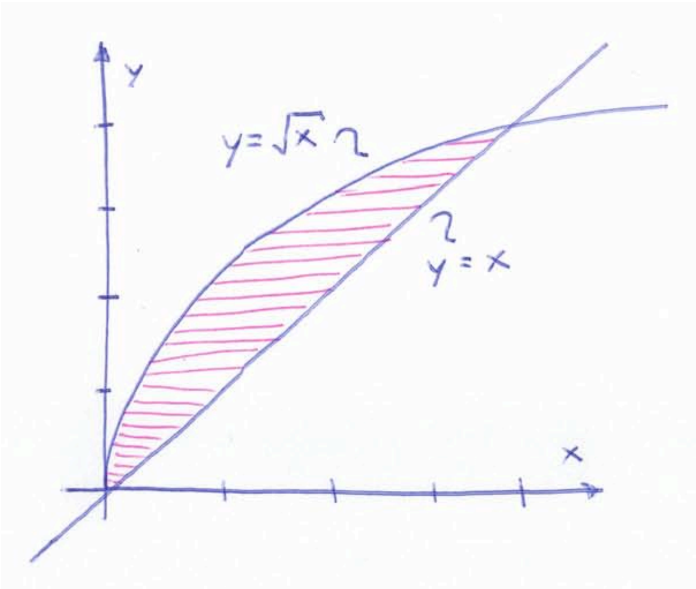
Integrate (3 marks)

$$\int \frac{x}{\sqrt{3x^2+1}} \; dx$$

Find the area of the shaded section (5 marks)



Find the area of the shaded section (5 marks)



(5 marks) Determine the dimensions of the fence to maximize the area given “L” meters of fencing. Parameters “a” and “L” should be set to:

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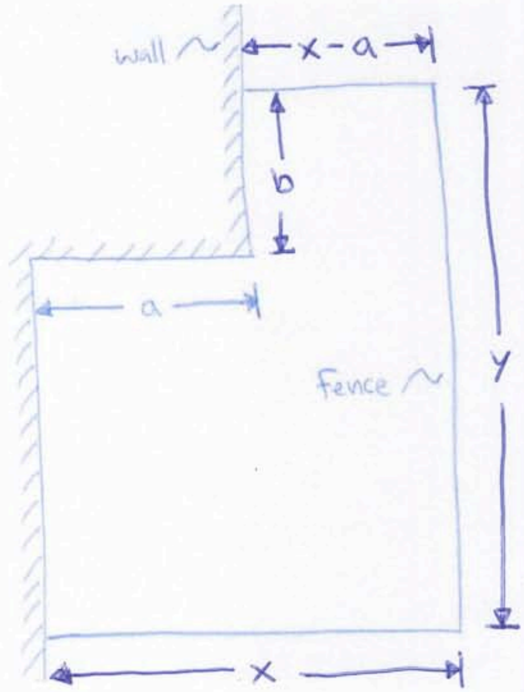


Perimeter

$$x + y + (x - a) = L$$

$$2x + y = L + a \qquad (1)$$

Area:

$$xy - ab = A \qquad (2)$$

Solve for y in (1)

$$y = L + a - 2x \qquad (1a)$$

sub (1a) into (2)

$$x\left[L + a - 2x\right] - ab = A$$

$$xL + ax - 2x^2 - ab = A$$

take derivative and set to zero (max)

$$L + a - 4x = 0$$

$$\boxed{x = \frac{L+a}{4}} \qquad (2a)$$

sub (2a) into (1a)

$$y = L + a - 2\left[\frac{L+a}{4}\right]$$

$$\boxed{y = \frac{L+a}{2}} \qquad (1b)$$

For a=0, L=20

x = 5, y = 10

For a = 2, L = 40

x = 10.5, y = 21

Integrate (2 marks)

$$= \int \frac{1}{2}x^2 + sin(2x) - \beta x \, dx$$

$$\boxed{= \frac{1}{6}x^3 - \frac{1}{2}\cos(2x) - \frac{1}{2}\beta x^2 + C}$$

Integrate (3 marks)

$$= \int \frac{x}{\sqrt{3x^2+1}} \, dx$$

let $u = 3x^2 + 1$

$$\frac{du}{dx} = 6x$$

$$dx = \frac{du}{6x}$$

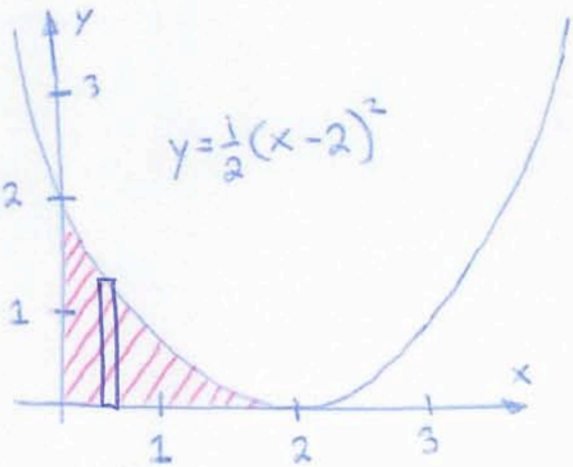
$$= \int \frac{\cancel{x}}{\sqrt{u}} \left[\frac{du}{6\cancel{x}}\right]$$

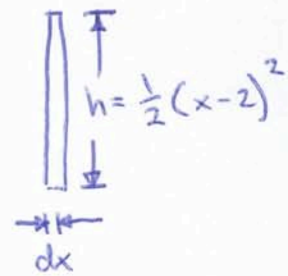
$$= \int \frac{1}{6} u^{-\frac{1}{2}} \, du$$

$$= \frac{1}{3} u^{\frac{1}{2}} + C$$

$$\boxed{= \frac{1}{3}\sqrt{3x^2+1} + C}$$

Find the area of the shaded section (5 marks)





$$dA = h\,dx$$

$$\int_0^2 dA = \int_0^2 \frac{1}{2}(x-2)^2\,dx$$

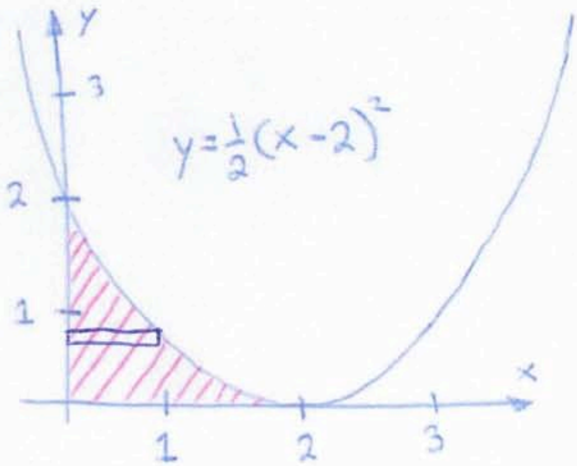
$$A = \left[\frac{1}{6}(x-2)^3\right]_0^2$$

$$= \left[\frac{1}{6}(2-2)^3\right] - \left[\frac{1}{6}(0-2)^3\right]$$

$$A = \frac{8}{6} = \frac{4}{3}$$

$$\boxed{A = \frac{4}{3} = 1.\bar{3}}$$

Find the area of the shaded section (5 marks)



solve for x and w:

$$y=\frac{1}{2}(x-2)^2$$

$$x-2=\pm\sqrt{2y}$$

$$x=2\pm\sqrt{2y}$$

$$w=2-\sqrt{2y}$$

so

$$dA = w\,dy$$

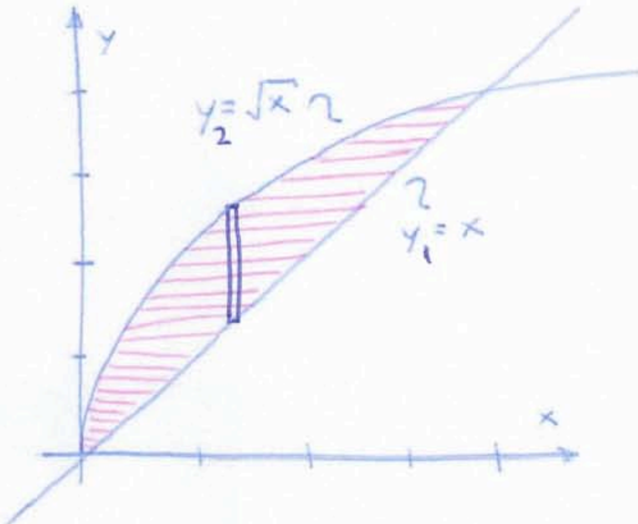
$$\int dA = \int_0^2 2-\sqrt{2y}\,dy$$

$$A=\left[2y-\frac{1}{3}(2y)^{3/2}\right]_0^2$$

$$=\left[2(2)-\frac{1}{3}(2(2))^{3/2}\right]-\left[2(0)+\frac{1}{3}(2(0))^{3/2}\right]$$

$$\boxed{A = 4-\frac{8}{3}=\frac{4}{3}=1.\overline{3}}$$

Find the area of the shaded section (5 marks)



Find intercept:

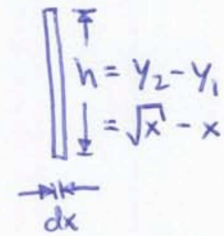
$y_1 = y_2$

$x = \sqrt{x}$

$\sqrt{x} = 1$

$\boxed{x = 1}$

Column:



$dA = h\,dx$

$$\int_0^1 dA = \int_0^1 \sqrt{x} - x \; dx$$

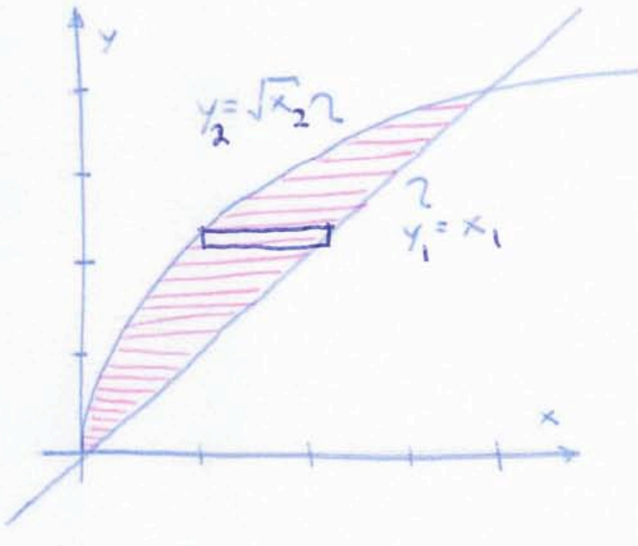
$$A = \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{2}x^2\right]_0^1$$

$$= \left[\frac{2}{3}(1)^{\frac{3}{2}} - \frac{1}{2}(1)^2\right] - \left[\frac{2}{3}(0)^{\frac{3}{2}} - \frac{1}{2}(0)^2\right]$$

$$= \frac{2}{3} - \frac{1}{2} = \frac{4}{6} - \frac{3}{6} = \frac{1}{6}$$

$$\boxed{A = \frac{1}{6} = 0.1\bar{6}}$$

Find the area of the shaded section (5 marks)



Find intercept:

$y_1 = y_2$

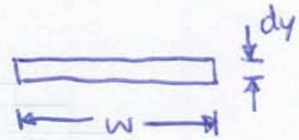
$x = \sqrt{x}$

$\sqrt{x} = 1$

$x = 1$

$y = 1$

row:



$w = x_1 - x_2$

solve for x and w:

$x_1 = y$

$x_2 = y^2$

$w = y - y^2$

so

$$dA = w\,dy$$

$$\int dA = \int_0^1 y - y^2\,dy$$

$$A = \left[\tfrac{1}{2}y^2 - \tfrac{1}{3}y^3\right]_0^1$$

$$= \left[\tfrac{1}{2}(1)^2 - \tfrac{1}{3}(1)^3\right] - \left[\tfrac{1}{2}(0)^2 - \tfrac{1}{3}(0)^3\right]$$

$$\boxed{A = \frac{1}{2} - \frac{1}{3} = \frac{1}{6} = 0.1\overline{6}}$$