

Instructor: Frank Secretain
Course: Math 2000
Date: December 11, 2025

Assessment: Test 4
Time allowed: 110 minutes
Devices allowed: Pencil, pen, eraser, calculator
Notes from instructor: Be neat. Show your work where needed. Box final answers.

Marks allocated: 3 question worth 20 marks
Percentage of final grade: 20% of final grade

Formula Sheet

Order of Operations

$$ac + bc = c(a + b)$$

exponent rules

$$a^n a^m = a^{n+m}$$

$$(a^n)^m = a^{nm}$$

$$(ab)^n = a^n b^n$$

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

$$a^{\frac{n}{m}} = \sqrt[m]{a^n}$$

logarithmic rules

$$\log_b(b^x) = x$$

$$b^{\log_b a} = a$$

$$\log_b x = \frac{\log(x)}{\log(b)}$$

$$\log(xy) = \log(x) + \log(y)$$

$$\log(x/y) = \log(x) - \log(y)$$

Forms of a 1st order polynomial

$$y = ax + b$$

Forms of a 2nd order polynomial

$$y = ax^2 + bx + c$$

(expanded form)

$$y = a(x - h)^2 + k$$

(vertex form)

$$y = a(x - m)(x - n)$$

(factored form)

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Definition of the derivative

$$\frac{d}{dx} f(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Euler's formula

$$e^{i\theta} = \cos \theta + i \sin \theta$$

Rules of differentiation

$$\frac{d}{dx} (fg) = \frac{d(f)}{dx} g + f \frac{d(g)}{dx}$$

(product rule)

$$\frac{d}{dx} \left(\frac{f}{g} \right) = \frac{\frac{d(f)}{dx} g - f \frac{d(g)}{dx}}{g^2}$$

(quotient rule)

$$\frac{d}{dx} (f(g)) = \frac{d(f)}{dx} \frac{d(g)}{dx}$$

(chain rule)

Derivatives of select functions

$$\frac{d}{dx} (ax^n) = anx^{n-1}$$

$$\frac{d}{dx} (\sin(x)) = \cos(x)$$

$$\frac{d}{dx} (\cos(x)) = -\sin(x)$$

$$\frac{d}{dx} (\tan(x)) = \frac{1}{(\cos(x))^2}$$

$$\frac{d}{dx} (a^x) = a^x \ln(a)$$

$$\frac{d}{dx} (\log_a x) = \frac{1}{x \ln(a)}$$

Integrals of select functions

$$\int ax^n dx = \begin{cases} \frac{a}{n+1} x^{n+1} & , n \neq -1 \\ \ln(|x|) & , n = -1 \end{cases} \quad (\text{polynomials})$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax)$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) \quad (\text{trigonometry})$$

$$\int \tan(ax) dx = \ln(|\sec(x)|)$$

$$\int a^x dx = \frac{1}{\ln(a)} a^x$$

(exponentials)

$$\int \ln(x) dx = x \ln(x) - x$$

(2 marks each) Solve for x in the following equations:

$$\frac{2 - 3^x}{2} = -7$$

$$\frac{\log(x^2 - 3)}{2} + 1 = 2$$

$$4.2 - \log_x(3) = 2$$

$$\frac{e-1}{e^{x+1}} + 1 = 2$$

(2 marks each) Evaluate to either rectangular or polar form:

$$z = 2i - 2i(i + 1) - 1$$

$$z = e^{\frac{3\pi i}{4}} (1 - i)i$$

$$z = (1 - i + e^{\frac{\pi i}{3}})^3$$

(2 marks each) Solve for z in either rectangular or polar form:

$$\frac{z}{2 + zi} + 1 = 0$$

$$z^3 = -10$$

$$\frac{1}{z} + z + 1 = 0$$

(2 marks each) Solve for x in the following equations:

$$\frac{2 - 3^x}{2} = -7$$

$$2 - 3^x = -14$$

$$3^x = 16$$

$$\log_3(3^x) = \log_3(16)$$

$$\begin{aligned} x &= \log_3(16) \\ &= \frac{\log(16)}{\log(3)} \\ &= 2.52 \end{aligned}$$

$$\frac{\log(x^2 - 3)}{2} + 1 = 2$$

$$\log(x^2 - 3) = 2$$

$$10^{\log(x^2 - 3)} = 10^2$$

$$x^2 - 3 = 100$$

$$x^2 = 103$$

$$x = \pm\sqrt{103}$$

$$\begin{aligned} x &= \sqrt{103} \\ &= 10.1 \end{aligned}$$

$$4.2 - \log_x(3) = 2$$

$$\log_x(3) = 4.2 - 2 = 2.2$$

$$x^{\log_x(3)} = x^{2.2}$$

$$3 = x^{2.2}$$

$$x = \sqrt[2.2]{3}$$

$$x = 1.65$$

$$\frac{e-1}{e^{x+1}} + 1 = 2$$

$$\frac{e-1}{e^{x+1}} = 1$$

$$e^{x+1} = e-1 = 1.718$$

$$\ln(e^{x+1}) = \ln(1.718)$$

$$x+1 = 0.541$$

$$x = -0.459$$

(2 marks each) Evaluate to either rectangular or polar form:

$$z = 2i - 2i(i + 1) - 1$$

$$z = 2i - 2i^2 - 2i - 1$$

$$z = -1 - 2(-1)$$

$$\begin{aligned} z &= 1 + 0i \\ &= 1e^{i0} \end{aligned}$$

$$z = e^{\frac{3\pi i}{4}}(1 - i)i$$

$$\begin{aligned} e^{\frac{3\pi i}{4}} &= \cos\left(\frac{3\pi}{4}\right) + i \sin\left(\frac{3\pi}{4}\right) \\ &= -0.707 + 0.707i \end{aligned}$$

$$= (-0.707 + 0.707i)(1 - i)i$$

$$= (-0.707 + 0.707i)(i + 1)$$

$$= \cancel{-0.707i} - 0.707 + 0.707i^2 + \cancel{0.707i}$$

$$= -1.414 + 0i$$

$$\begin{aligned} z &= -1.414 + 0i \\ &= 1.414 e^{i\pi} \end{aligned}$$

$$z = (1 - i + e^{\frac{\pi i}{3}})^3$$

$$= (1 - i + [0.5 + 0.866i])^3$$

$$= (1.5 - 0.134i)^3$$

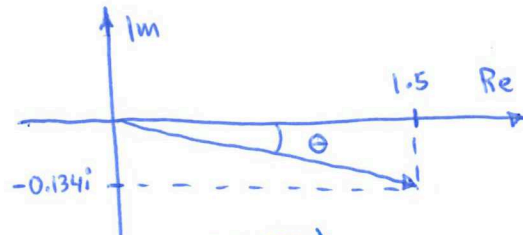
$$= (1.51 e^{-0.089i})^3$$

$$= 1.51^3 e^{-}$$

$$= 3.41 e^{-0.267i}$$

$$e^{\frac{\pi i}{3}} = \cos\left(\frac{\pi}{3}\right) + i \sin\left(\frac{\pi}{3}\right)$$

$$= 0.5 + 0.866i$$



$$\theta = \tan^{-1}\left(\frac{0.134}{1.5}\right) = 0.089$$

$$R = \sqrt{(1.5)^2 + (-0.134)^2} = 1.51$$

$$z = 3.41 e^{-0.267i}$$

$$= 3.29 - 0.90i$$

(2 marks each) Solve for z in either rectangular or polar form:

$$\frac{z}{2+zi} + 1 = 0$$

$$\frac{z}{2+iz} = -1$$

$$z = -2 - iz$$

$$z + iz = -2$$

$$z(1+i) = -2$$

$$z = \frac{-2}{(1+i)} \frac{(1-i)}{(1-i)}$$

$$= \frac{-2 + 2i}{1 - i + i - i^2}$$

$$= \frac{-2 + 2i}{2} = -1 + i$$

$$\begin{aligned} z &= -1 + i \\ &= \sqrt{2} e^{\frac{3\pi}{4}i} \\ &= 1.41 e^{2.36i} \end{aligned}$$

$$z^3 = -10$$

$$z = (-10)^{\frac{1}{3}}$$

$$\theta = \pi$$

$$z = (10 e^{i\pi})^{\frac{1}{3}} = 10^{\frac{1}{3}} e^{\frac{\pi}{3}i} =$$

$$10^{\frac{1}{3}} e^{1.05i} = 1.08 + 1.87i$$

$$\theta = \pi + 2\pi$$

$$z = (10 e^{3\pi i})^{\frac{1}{3}} = 10^{\frac{1}{3}} e^{i\pi} =$$

$$\sqrt[3]{10} e^{3.14i} = -2.15 + 0i$$

$$\theta = \pi + 4\pi$$

$$z = (10 e^{5\pi i})^{\frac{1}{3}} = 10^{\frac{1}{3}} e^{\frac{5\pi}{3}i} =$$

$$2.15 e^{5.24i} = 1.08 - 1.87i$$

$$\frac{1}{z} + z + 1 = 0$$

$$1 + z^2 + z = 0$$

$$z^2 + z + 1 = 0$$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)}$$

$$= -\frac{1}{2} \pm \frac{\sqrt{-3}}{2}$$

$$z = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$$

$$= -0.5 \pm 0.866i$$

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$$\int \tan(ax) dx = \ln(|\sec(x)|)$$

$$\int a^x dx = \frac{1}{\ln(a)} a^x$$

(exponentials)

$$\int \ln(x) dx = x \ln(x) - x$$

(8 marks) Solve for x in the following equations:

$$\frac{1 + 4^x}{2} = 3$$

$$\frac{\ln(x^2 - 1)}{4} + 1 = 2$$

$$1 + \log_x(2) = 3$$

$$\frac{e^{x+1}}{e+1} = 1$$

(6 marks) Evaluate to either rectangular or polar form:

$$z = i + i(2 + i) + 2$$

$$z = (2 - i)e^{\frac{\pi}{4}i}$$

$$z = (2 + i - e^{\frac{\pi}{2}i})^3$$

(6 marks) Solve for z in either rectangular or polar form:

$$\frac{3z + 2}{1 + zi} + 2 = 0$$

$$z^3 = 27$$

$$z + \frac{1}{z + 1} = 0$$

(4 marks) Solve for x in the following equations:

$$\frac{1 + 4^x}{2} = 3$$

$$1 + 4^x = 6$$

$$4^x = 5$$

$$\log_4(4^x) = \log_4(5)$$

$$x = \log_4(5)$$

$$= \frac{\log(5)}{\log(4)} = 1.16$$

$$\frac{\ln(x^2 - 1)}{4} + 1 = 2$$

$$\ln(x^2 - 1) = 4$$

$$e^{\ln(x^2 - 1)} = e^4$$

$$x^2 - 1 = e^4$$

$$x^2 = e^4 + 1$$

$$x = \sqrt{e^4 + 1}$$

$$x = \sqrt{e^4 + 1}$$

$$= 7.46$$

$$1 + \log_x(2) = 3$$

$$\log_x(2) = 2$$

$$x^{\log_x(2)} = x^2$$

$$2 = x^2$$

$$x = \pm\sqrt{2}$$

$$x = \sqrt{2}$$

$$\frac{e^{x+1}}{e+1} = 1$$

$$e^{x+1} = e+1$$

$$\ln(e^{x+1}) = \ln(e+1)$$

$$x+1 = \ln(e+1)$$

$$x = \ln(e+1) - 1$$

$$\begin{aligned} x &= \ln(e+1) - 1 \\ &= 0.313 \end{aligned}$$

(4 marks) Evaluate to either rectangular or polar form:

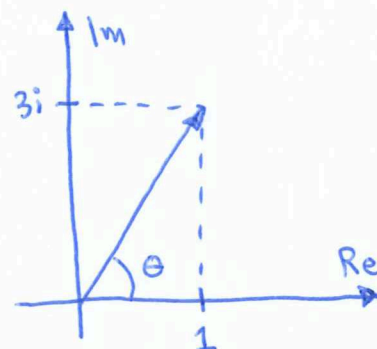
$$z = i + i(2 + i) + 2$$

$$= i + 2i + i^2 + 2$$

$$= 3i - 1 + 2$$

$$= 1 + 3i$$

$$\begin{aligned} Z &= 1 + 3i \\ &= \sqrt{10} e^{1.25i} \\ &= 3.16 e^{1.25i} \end{aligned}$$



$$\theta = \tan^{-1}\left(\frac{3}{1}\right) = 1.25$$

$$R = \sqrt{1^2 + 3^2} = \sqrt{10}$$

$$z = (2 - i)e^{\frac{\pi}{4}i}$$

$$= (2 - i)(0.707 + 0.707i)$$

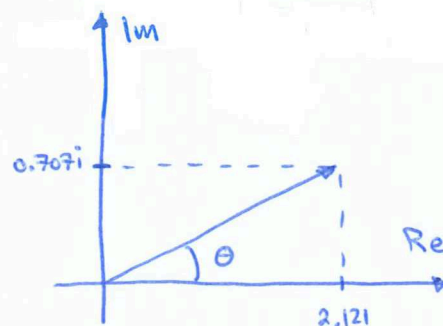
$$= 1.414 + 1.414i - 0.707i - 0.707i^2$$

$$= 0.707i + 1.414 + 0.707$$

$$= 2.121 + 0.707i$$

$$\begin{aligned} Z &= 2.121 + 0.707i \\ &= 2.24 e^{0.322i} \end{aligned}$$

$$\begin{aligned} e^{\frac{\pi}{4}i} &= \cos\left(\frac{\pi}{4}\right) + i \sin\left(\frac{\pi}{4}\right) \\ &= 0.707 + 0.707i \end{aligned}$$



$$\theta = \tan^{-1}\left(\frac{0.707}{2.121}\right) = 0.322$$

$$R = \sqrt{2.121^2 + 0.707^2} = 2.24$$

$$z = (2 + i - e^{\frac{\pi}{2}i})^3$$

$$z = (2 + i - [i])^3$$

$$= (2)^3$$

$$= 8$$

$$e^{\frac{\pi}{2}i} = \cos\left(\frac{\pi}{2}\right) + i \sin\left(\frac{\pi}{2}\right)$$

$$= 0 + i$$

$$\boxed{z = 8}$$

(6 marks) Solve for z in either rectangular or polar form:

$$\frac{3z+2}{1+zi} + 2 = 0$$

$$3z+2 = -2(1+zi)$$

$$3z+2 = -2-2zi$$

$$3z+2zi = -4$$

$$z(3+2i) = -4$$

$$z = \frac{(-4)}{(3+2i)} \frac{(3-2i)}{(3-2i)}$$

$$z = \frac{-12+8i}{9-6i+6i-4i^2}$$

$$z = \frac{-12+8i}{13}$$

$$z = -\frac{12}{13} + \frac{8}{13}i$$

$$z^3 = 27$$

$$z = (27)^{\frac{1}{3}}$$

$$\theta=0$$

$$z = (27e^{i0})^{\frac{1}{3}} = 27^{\frac{1}{3}} e^{\frac{0}{3}i} = 3e^{0i} = 3+0i$$

$$\theta=2\pi$$

$$z = (27e^{2\pi i})^{\frac{1}{3}} = 27^{\frac{1}{3}} e^{\frac{2\pi}{3}i} = 3e^{\frac{2\pi}{3}i} = -1.5 + 2.6i$$

$$\theta=4\pi$$

$$z = (27e^{4\pi i})^{\frac{1}{3}} = 27^{\frac{1}{3}} e^{\frac{4\pi}{3}i} = 3e^{\frac{4\pi}{3}i} = -1.5 - 2.6i$$

$$z + \frac{1}{z+1} = 0$$

$$z(z+1) + 1 = 0(z+1)$$

$$z^2 + z + 1 = 0$$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1^2 - 4}}{2}$$

$$= -\frac{1}{2} \pm \frac{\sqrt{-3}}{2}$$

$$= -0.5 \pm 0.87i$$

$$z = -0.5 \pm 0.87i$$