

Instructor: Frank Secretain
Course: Math 2000
Date: October 16, 2025

Assessment: Test 2
Time allowed: 110 minutes
Devices allowed: Pencil, pen, eraser, calculator
Notes from instructor: Be neat. Show your work where needed. Box final answers.

Marks allocated: 3 question worth 20 marks
Percentage of final grade: 20% of final grade

Formula Sheet

Order of Operations

$$ac + bc = c(a + b)$$

exponent rules

$$a^n a^m = a^{n+m}$$

$$(a^n)^m = a^{nm}$$

$$(ab)^n = a^n b^n$$

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

$$a^{\frac{n}{m}} = \sqrt[m]{a^n}$$

logarithmic rules

$$\log_b(b^x) = x$$

$$b^{\log_b a} = a$$

$$\log_b x = \frac{\log(x)}{\log(b)}$$

$$\log(xy) = \log(x) + \log(y)$$

$$\log(x/y) = \log(x) - \log(y)$$

Forms of a 1st order polynomial

$$y = ax + b$$

Forms of a 2nd order polynomial

Quadratic Formula

$$y = ax^2 + bx + c \quad (\text{expanded form})$$

$$y = a(x - h)^2 + k \quad (\text{vertex form})$$

$$y = a(x - m)(x - n) \quad (\text{factored form})$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Definition of the derivative

$$\frac{d}{dx} f(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Rules of differentiation

$$\frac{d}{dx}(fg) = \frac{d(f)}{dx}g + f\frac{d(g)}{dx}$$

(product rule)

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{\frac{d(f)}{dx}g - f\frac{d(g)}{dx}}{g^2}$$

(quotient rule)

$$\frac{d}{dx}(f(g)) = \frac{d(f)}{dx} \frac{d(g)}{dx}$$

(chain rule)

Derivatives of select functions

$$\frac{d}{dx}(ax^n) = anx^{n-1}$$

$$\frac{d}{dx}(\sin(x)) = \cos(x)$$

$$\frac{d}{dx}(\cos(x)) = -\sin(x)$$

$$\frac{d}{dx}(\tan(x)) = \frac{1}{(\cos(x))^2}$$

$$\frac{d}{dx}(a^x) = a^x \ln(a)$$

$$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln(a)}$$

Integrals of select functions

$$\int ax^n dx = \begin{cases} \frac{a}{n+1} x^{n+1} & , n \neq -1 \\ \ln(|x|) & , n = -1 \end{cases} \quad (\text{polynomials})$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax)$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) \quad (\text{trigonometry})$$

$$\int \tan(ax) dx = \ln(|\sec(x)|)$$

$$\int a^x dx = \frac{1}{\ln(a)} a^x$$

(exponentials)

$$\int \ln(x) dx = x \ln(x) - x$$

(10 marks) Solve for x in the following equations:

$$5^{x+1} = 23$$

$$\log_3(x) = 0.45$$

$$\frac{3\ln(x+1)-2}{3}+\Delta^2=\epsilon$$

$$\frac{e-1}{e^{x+1}}=1$$

(5 marks) You run 410 m North, 230 m at 15° North of East and 220 m at 10° East of South. How far are you from where you started?

(5 marks) Convert to Polar form:

$$z = (2 + i)(2i - 3)$$

$$z = \frac{2 + i}{2i - 3}$$

(10 marks) Solve for x in the following equations:

$$5^{x+1} = 23$$

$$\log_5(5^{x+1}) = \log_5(23)$$

$$x+1 = \log_5(23)$$

$$\begin{aligned} x &= \log_5(23) - 1 \\ &= \frac{\log(23)}{\log(5)} - 1 \\ &= 0.948 \end{aligned}$$

$$\log_3(x) = 0.45$$

$$3^{\log_3(x)} = 3^{0.45}$$

$$\begin{aligned} x &= 3^{0.45} \\ &= 1.64 \end{aligned}$$

$$\frac{3\ln(x+1)-2}{3} + \Delta^2 = e^{-\Delta^2}$$

$$3 \left(\frac{3\ln(x+1)-2}{3} \right) = (e - \Delta^2) 3$$

$$3\ln(x+1)-2 = 3(e - \Delta^2)$$

$$\frac{3\ln(x+1)}{3} = \frac{3(e - \Delta^2) + 2}{3}$$

$$\ln(x+1) = \frac{1}{3}[3(e - \Delta^2) + 2]$$

$$e^{\ln(x+1)} = e^{\frac{1}{3}[3(e - \Delta^2) + 2]}$$

$$x+1 = e^{\frac{1}{3}[3(e - \Delta^2) + 2]}$$

$$x = e^{\frac{1}{3}[3(e - \Delta^2) + 2]} - 1$$

$$\frac{e-1}{e^{x+1}} = 1$$

$$e^{x+1} \left(\frac{e-1}{e^{x+1}} \right) = (1) e^{x+1}$$

$$e-1 = e^{x+1}$$

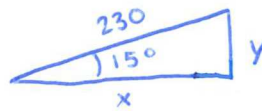
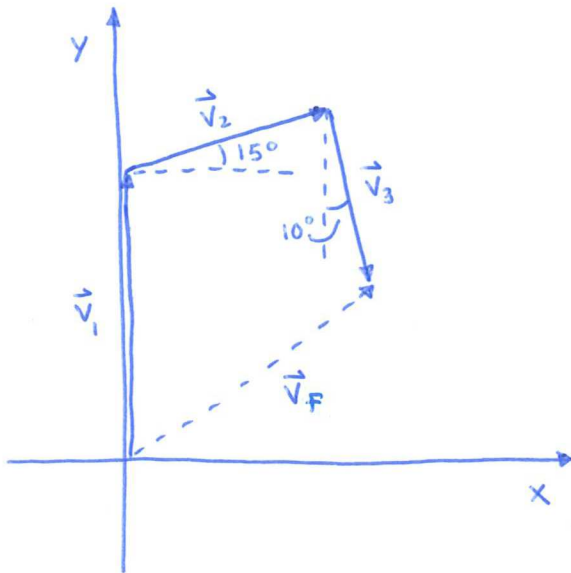
$$\ln(e-1) = \ln(e^{x+1})$$

$$\ln(e-1) = x+1$$

$$x = \ln(e-1) - 1$$

$$= -0.459$$

(5 marks) You run 410 m North, 230 m at 15° North of East and 220 m at 10° East of South. How far are you from where you started?

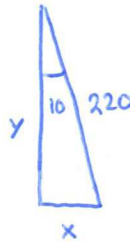


$$x = 230 \cos(15)$$

$$= 222.16$$

$$y = 230 \sin(15)$$

$$= 59.53$$



$$x = 220 \sin(10)$$

$$= 38.20$$

$$y = 220 \cos(10)$$

$$= 216.66$$

$$\vec{V}_1 = 0 \hat{x} + 410 \hat{y}$$

$$\vec{V}_2 = 222.16 \hat{x} + 59.53 \hat{y}$$

$$+ \vec{V}_3 = 38.20 \hat{x} - 216.66 \hat{y}$$

$$\vec{V}_F = 260.36 \hat{x} + 252.87 \hat{y}$$

$$|\vec{V}_F| = \sqrt{(260.36)^2 + (252.87)^2}$$

$$= 362.95$$

$|\vec{V}_F| = 363 \text{ m}$

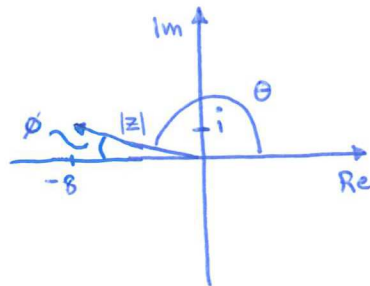
(5 marks) Convert to Polar form:

$$z = (2 + i)(2i - 3)$$

$$= 4i - 6 + 2i^2 - 3i$$

$$= -6 + 2[-1] + i$$

$$= -8 + i$$



$$|z| = \sqrt{(-8)^2 + (1)^2} = 8.06$$

$$\phi = \tan^{-1}\left(\frac{1}{8}\right) = 7.13^\circ$$

$$= 0.12 \text{ rads}$$

$$\theta = \pi - 0.12 = 3.02 \text{ rads}$$

so

$$z = 8.06 e^{3.02i}$$

$$z = \frac{2 + i}{2i - 3}$$

$$= \frac{(2 + i)(2i + 3)}{(2i - 3)(2i + 3)}$$

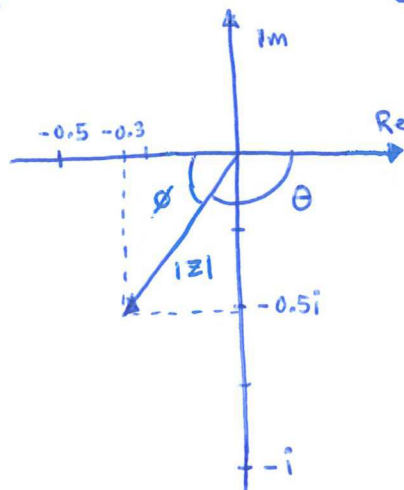
$$= \frac{4i + 6 + 2i^2 + 3i}{4i^2 + \cancel{6i} - \cancel{6i} - 9}$$

$$= \frac{6 + 2[-1] + 7i}{4[-1] - 9}$$

$$= \frac{4 + 7i}{-13}$$

$$= -\frac{4}{13} - \frac{7}{13}i$$

$$= -0.308 - 0.538i$$



$$|z| = \sqrt{(-0.308)^2 + (-0.538)^2}$$

$$= 0.620$$

$$\phi = \tan^{-1}\left(\frac{0.538}{0.308}\right) = 60.2^\circ$$

$$= 1.05 \text{ rad}$$

$$\theta = -\pi + 1.05 = -2.09 \text{ rads}$$

so

$$z = 0.620 e^{-2.09i}$$

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$$\int a^x dx = \frac{1}{\ln(a)} a^x$$

(exponentials)

$$\int \ln(x) dx = x \ln(x) - x$$

(10 marks each) Solve for x in the following equations:

$$4^x = 20$$

$$\log(x) = 0.23$$

$$\frac{\ln(x+1)-1}{b-a}+\Gamma=\alpha$$

$$\frac{e^{x+1}}{e+1}=1$$

(5 marks) You run 810 m East, 250 m at 35° North of East and 220 m at 40° West of North. How far are you from where you started?

(5 marks) Convert to Polar form:

$$z = 3 + 5i$$

$$z = \frac{3 - i}{1 - i}$$

(10 marks each) Solve for x in the following equations:

$$4^x = 20$$

$$\log_4(4^x) = \log_4(20)$$

$$x = \log_4(20) = \frac{\log(20)}{\log(4)} = 2.16$$

$$\boxed{x = 2.2}$$

$$\log(x) = 0.23$$

$$10^{\log(x)} = 10^{0.23}$$

$$x = 1.698$$

$$\boxed{x = 1.7}$$

$$\frac{\ln(x+1) - 1}{b-a} + \overset{-1}{\Gamma} = \overset{-1}{\alpha}$$

$$(b-a) \left(\frac{\ln(x+1) - 1}{b-a} \right) = (\alpha - 1)(b-a)$$

$$\ln(x+1) - 1 = (\alpha - 1)(b-a)$$

$$\ln(x+1) = (\alpha - 1)(b-a) + 1$$

$$e^{\ln(x+1)} = e^{(\alpha - 1)(b-a) + 1}$$

$$x+1 = e^{(\alpha - 1)(b-a) + 1}$$

$$x = e^{(\alpha - 1)(b-a) + 1} - 1$$

$$\frac{e^{x+1}}{e+1} = 1$$

$$e^{x+1} = e+1$$

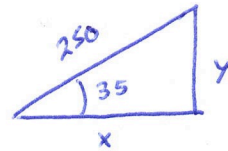
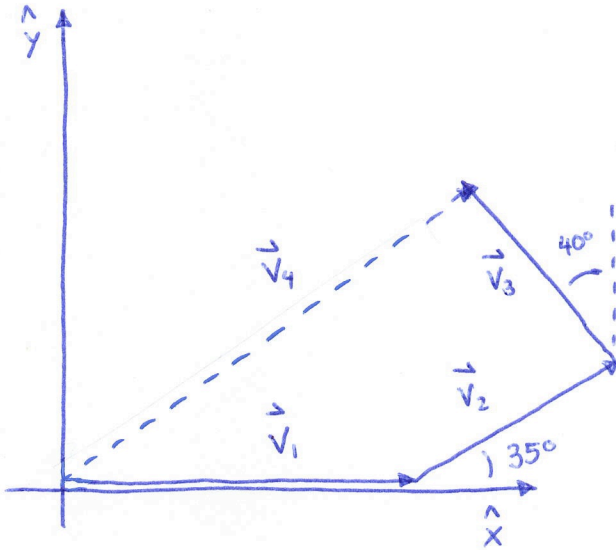
$$\ln(e^{x+1}) = \ln(e+1)$$

$$x+1 = \ln(e+1)$$

$$x = \ln(e+1) - 1$$

$$= 0.31$$

(5 marks) You run 810 m East, 250 m at 35° North of East and 220 m at 40° West of North. How far are you from where you started?

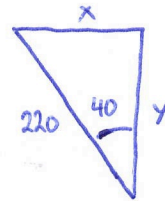


$$x = 250 \cos(35)$$

$$= 204.79$$

$$y = 250 \sin(35)$$

$$= 143.39$$



$$x = 220 \sin(40)$$

$$= 141.41$$

$$y = 220 \cos(40)$$

$$= 168.53$$

$$\vec{V}_1 = 810 \hat{x} + 0 \hat{y}$$

$$\vec{V}_2 = 204.79 \hat{x} + 143.39 \hat{y}$$

$$+ \vec{V}_3 = -141.41 \hat{x} + 168.53 \hat{y}$$

$$\vec{V}_F = 873.38 \hat{x} + 311.92 \hat{y}$$

$$|\vec{V}_F| = \sqrt{(873.38)^2 + (311.92)^2}$$

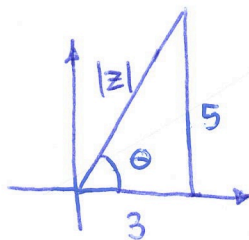
$$|\vec{V}_F| = 927.41$$

$|\vec{V}_F| = 927 \text{ m}$

(5 marks) Convert to Polar form:

$$z = 3 + 5i$$

$$z = 5.83 e^{1.03i}$$



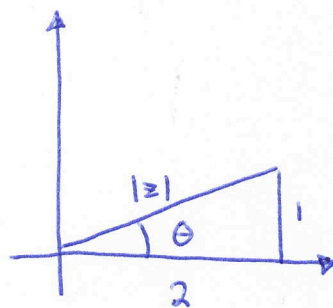
$$|z| = \sqrt{3^2 + 5^2} = 5.83$$

$$\theta = \tan^{-1}\left(\frac{5}{3}\right) = 59.0^\circ$$
$$= 1.03 \text{ rads}$$

$$z = \frac{3-i}{1-i}$$

$$z = \frac{(3-i)(1+i)}{(1-i)(1+i)} = \frac{3 + 3i - i + 1}{1 + i - i + 1} = \frac{4 + 2i}{2} = 2 + i$$

$$z = 2.24 e^{0.46i}$$



$$|z| = \sqrt{2^2 + 1^2} = \sqrt{5} = 2.24$$

$$\theta = \tan^{-1}\left(\frac{1}{2}\right) = 26.6^\circ$$
$$= 0.46 \text{ rads}$$