Instructor:	Frank Secretain
Course:	Math 7
Date:	December 11, 2025
Assessment:	Test 4
Time allowed:	110 minutes
Devices allowed:	Pencil, pen, eraser, calculator
Notes from instructor:	Be neat. Show your work where needed. Box final answers.
Marks allocated:	4 question worth 20 marks
Percentage of final grade:	20% of final grade

DERIVATIVE RULES

Constant rule	$\frac{d}{dx}(C) = 0$
Multiplier rule	$\frac{d}{dx}\left(C \cdot f(x)\right) = C \cdot f'(x)$
Sum/difference rule	$\frac{d}{dx}\left(f(x) \pm g(x)\right) = f'(x) \pm g'(x)$
Product rule	$\frac{d}{dx}\left(f(x) \cdot g(x)\right) = f'(x)g(x) + f(x)g'(x)$
Quotient rule	$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - g'(x)f(x)}{\left(g(x)\right)^2}$
Chain rule	$\frac{d}{dx}\left(f(g(x)\right) = f'\left(g(x)\right) \cdot g'(x)$
Power rule	$\frac{d}{dx}(x^n) = nx^{n-1}$
Exponent rule	$\frac{d}{dx}(e^x) = e^x$ $\frac{d}{dx}(b^x) = b^x \cdot \ln b$
Logarithm rule	$\frac{d}{dx}(\ln x) = \frac{1}{x}$ $\frac{d}{dx}(\log_b x) = \frac{1}{x} \cdot \frac{1}{\ln b}$
Trigonometric rules	$\frac{d}{dx}(\sin x) = \cos x$ $\frac{d}{dx}(\cos x) = -\sin x$

Formula Sheet

Order of Operations

$$ac + bc = c(a+b)$$

exponent rules

$$a^n a^m = a^{n+m} \qquad (a^n)^m = a^{nm} \qquad (ab)^n = a^n b^n$$

$$a^0 = 1 \qquad a^{-n} = \frac{1}{a^n} \qquad a^{\frac{n}{m}} = \sqrt[m]{a^n}$$

logarithmic rules

$$\log_b(b^x) = x \qquad b^{\log_b a} = a \qquad \log_b x = \frac{\log(x)}{\log(b)}$$

$$\log(xy) = \log(x) + \log(y) \qquad \log(x/y) = \log(x) - \log(y)$$

Forms of a 1st order polynomial

$$y = ax + b$$

Forms of a 2nd order polynomial

$$y = ax^2 + bx + c$$ (expanded form)

$$y = a(x-h)^2 + k$$ (vertex form)

$$y = a(x-m)(x-n)$$ (factored form)

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Definition of the derivative

$$\frac{d}{dx} f(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Euler's formula

$$e^{i\theta} = \cos\theta + i \sin\theta$$

Rules of differentiation

$$\frac{\mathrm{d}}{\mathrm{dx}}(fg) = \frac{\mathrm{d}(f)}{\mathrm{dx}} g + f \frac{\mathrm{d}(g)}{\mathrm{dx}}$$

(product rule)

$$\frac{\mathrm{d}}{\mathrm{dx}}\left(\frac{f}{g}\right) = \frac{\frac{\mathrm{d}(f)}{\mathrm{dx}} g - f \frac{\mathrm{d}(g)}{\mathrm{dx}}}{g^2}$$

(quotient rule)

$$\frac{\mathrm{d}}{\mathrm{dx}}(f(g)) = \frac{\mathrm{d}(f)}{\mathrm{dx}} \frac{\mathrm{d}(g)}{\mathrm{dx}}$$

(chain rule)

Derivatives of select functions

$$\frac{d}{dx}(ax^n) = anx^{n-1}$$

$$\frac{d}{dx}(sin(x)) = cos(x)$$

$$\frac{d}{dx}(cos(x)) = -sin(x)$$

$$\frac{d}{dx}(tan(x)) = \frac{1}{(cos(x))^2}$$

$$\frac{d}{dx}(a^x) = a^x ln(a)$$

$$\frac{d}{dx}(log_a x) = \frac{1}{x ln(a)}$$

Integrals of select functions

$$\int ax^n dx = \left\{ \begin{array}{ll} \frac{a}{n+1} x^{n+1} & , n \neq -1 \\ ln(|x|) & , n = -1 \end{array} \right\}$$ (polynomials)

$$\int sin(ax) dx = -\frac{1}{a} cos(ax)$$

$$\int cos(ax) dx = \frac{1}{a} sin(ax)$$ (trigonometry)

$$\int tan(ax) dx = ln(|sec(x)|)$$

$$\int a^x dx = \frac{1}{ln(a)} a^x$$ (exponentials)

$$\int ln(x) dx = x ln(x) - x$$

(2 marks each) Solve for x in the following equations:

$$\frac{2 - 3^x}{2} = -7$$

$$\frac{log(x^2 - 3)}{2} + 1 = 2$$

(2 marks each) Evaluate to either rectangular or polar form:

$$z = 2i - 2i(i + 1) - 1$$

$$z = e^{\frac{3\pi i}{4}}(1 - i)i$$

(2 marks each) Solve for z in either rectangular or polar form:

$$\frac{z}{2+zi}+1=0$$

$$z^3=-10$$

$$\frac{1}{z} + z + 1 = 0$$

(2 marks each) Determine the derivative with respect to x of the following equations.

$$y = 3x^{\frac{2}{3}} - \sin(x)$$

$$y = 3x^{\frac{2}{3}} \sin(x)$$

$$y^2x - x + y = \sin(y)$$

(2 marks each) Solve for x in the following equations:

$$\frac{2-3^x}{2} = -7$$

$2 - 3^x = -14$

$3^x = 16$

$\log_3(3^x) = \log_3(16)$

$$\boxed{\begin{aligned} x &= \log_3(16) \\ &= \frac{\log(16)}{\log(3)} \\ &= 2.52 \end{aligned}}$$

$$\frac{log(x^2-3)}{2} + 1 = 2$$

$\log(x^2-3) = 2$

$10^{\log(x^2-3)} = 10^2$

$x^2 - 3 = 100$

$x^2 = 103$

$x = \pm\sqrt{103}$

$$\boxed{\begin{aligned} x &= \sqrt{103} \\ &= 10.1 \end{aligned}}$$

(2 marks each) Evaluate to either rectangular or polar form:

$$z = 2i - 2i(i+1) - 1$$

$z = 2i - 2i^2 - 2i - 1$

$z = -1 - 2(-1)$

$$\boxed{\begin{aligned} z &= 1 + 0i \\ &= 1e^{i0} \end{aligned}}$$

$$z = e^{\frac{3\pi i}{4}}(1-i)i$$

$e^{\frac{3\pi}{4}i} = \cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)$

$= -0.707 + 0.707i$

$= (-0.707 + 0.707i)(1-i)i$

$= (-0.707 + 0.707i)(i+1)$

$= \cancel{-0.707i} - 0.707 + 0.707i^2 + \cancel{0.707i}$

$= -1.414 + 0i$

$$\boxed{\begin{aligned} z &= -1.414 + 0i \\ &= 1.414\,e^{i\pi} \end{aligned}}$$

(2 marks each) Solve for z in either rectangular or polar form:

$$\frac{z}{2+zi}+1=0$$

$$\frac{z}{2+iz} = -1$$

$$z = -2 - iz$$

$$z + iz = -2$$

$$z(1+i) = -2$$

$$z = \frac{-2}{(1+i)}\frac{(1-i)}{(1-i)}$$

$$= \frac{-2+2i}{1-i+i-i^2}$$

$$= \frac{-2+2i}{2} = -1+i$$

$$\boxed{\begin{aligned} z &= -1+i \\ &= \sqrt{2}\, e^{\frac{3\pi}{4}i} \\ &= 1.41\, e^{2.36i} \end{aligned}}$$

$$z^3 = -10$$

$$z = (-10)^{\frac{1}{3}}$$

$\theta = \pi$ $\quad z = \left(10e^{i\pi}\right)^{\frac{1}{3}} = 10^{\frac{1}{3}} e^{\frac{\pi}{3}i} = 10^{\frac{1}{3}} e^{1.05i} = 1.08 + 1.87i$

$\theta = \pi + 2\pi$ $\quad z = \left(10e^{3\pi i}\right)^{\frac{1}{3}} = 10^{\frac{1}{3}} e^{i\pi} = \sqrt[3]{10}\, e^{3.14i} = -2.15 + 0i$

$\theta = \pi + 4\pi$ $\quad z = \left(10e^{5\pi i}\right)^{\frac{1}{3}} = 10^{\frac{1}{3}} e^{\frac{5\pi}{3}i} = 2.15\, e^{5.24i} = 1.08 - 1.87i$

$$\frac{1}{z} + z + 1 = 0$$

$$1 + z^2 + z = 0$$

$$z^2 + z + 1 = 0$$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)}$$

$$= -\frac{1}{2} \pm \frac{\sqrt{-3}}{2}$$

$$\boxed{\begin{aligned} z &= -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i \\ &= -0.5 \pm 0.866 i \end{aligned}}$$

(2 marks each) Determine the derivative with respect to x of the following equations.

$$y = 3x^{\frac{2}{3}} - \sin(x)$$

$\frac{d}{dx}($

$$\boxed{\frac{dy}{dx} = 2x^{-\frac{1}{3}} - \cos(x)}$$

$$y = 3x^{\frac{2}{3}} \sin(x)$$

$\frac{d}{dx}($

$$\boxed{\frac{dy}{dx} = 2x^{-\frac{1}{3}} \sin(x) + 3x^{\frac{2}{3}} \cos(x)}$$

$$y^2x - x + y = \sin(y)$$

$\frac{d}{dx}$ ↓

$$\boxed{2yx\frac{dy}{dx} + y^2 - 1 + \frac{dy}{dx} = \cos(y)\frac{dy}{dx}}$$

$$\frac{dy}{dx}\left(2yx + 1 - \cos(y)\right) = 1 - y^2$$

$$\boxed{\frac{dy}{dx} = \frac{1 - y^2}{2yx + 1 - \cos(y)}}$$

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$$\int a^x dx = \frac{1}{ln(a)} a^x$$ (exponentials)

$$\int ln(x) dx = xln(x) - x$$

(4 marks) Solve for x in the following equations:

$$\frac{1+4^x}{2} = 3$$

$$\frac{\ln(x^2-1)}{4} + 1 = 2$$

(4 marks) Evaluate to either rectangular or polar form:

$$z = i + i(2 + i) + 2$$

$$z = (2 - i)e^{\frac{\pi}{4}i}$$

(6 marks) Solve for z in either rectangular or polar form:

$$\frac{3z+2}{1+zi}+2=0$$

$$z^3=27$$

$$z + \frac{1}{z+1} = 0$$

(6 marks) Determine the derivative with respect to x of the following equations.

$$y = \frac{4x^3}{7} + 2x^{\frac{3}{4}}$$

$$y = x^2 \sin(x^{a+1} - bx + c) - \frac{1}{x}$$

$$y^2x - x = y\sin(x) + 2$$

(4 marks) Solve for x in the following equations:

$$\frac{1+4^x}{2} = 3$$

$1 + 4^x = 6$

$4^x = 5$

$\log_4(4^x) = \log_4(5)$

$$\boxed{\begin{aligned} x &= \log_4(5) \\ &= \frac{\log(5)}{\log(4)} = 1.16 \end{aligned}}$$

$$\frac{\ln(x^2-1)}{4} + 1 = 2$$

$\ln(x^2-1) = 4$

$e^{\ln(x^2-1)} = e^4$

$x^2 - 1 = e^4$

$x^2 = e^4 + 1$

$x = \sqrt{e^4+1}$

$$\boxed{\begin{aligned} x &= \sqrt{e^4+1} \\ &= 7.46 \end{aligned}}$$

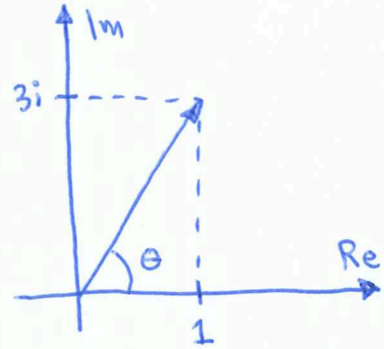
(4 marks) Evaluate to either rectangular or polar form:

$$z = i + i(2+i) + 2$$

$$= i + 2i + i^2 + 2$$

$$= 3i - 1 + 2$$

$$= 1 + 3i$$



$$\theta = \tan^{-1}\left(\tfrac{3}{1}\right) = 1.25$$

$$R = \sqrt{1^2 + 3^2} = \sqrt{10}$$

$$\boxed{\begin{aligned} Z &= 1 + 3i \\ &= \sqrt{10}\, e^{1.25i} \\ &= 3.16\, e^{1.25i} \end{aligned}}$$

$$z = (2 - i)e^{\frac{\pi}{4}i}$$

$$e^{\frac{\pi}{4}i} = \cos\left(\tfrac{\pi}{4}\right) + i\sin\left(\tfrac{\pi}{4}\right)$$

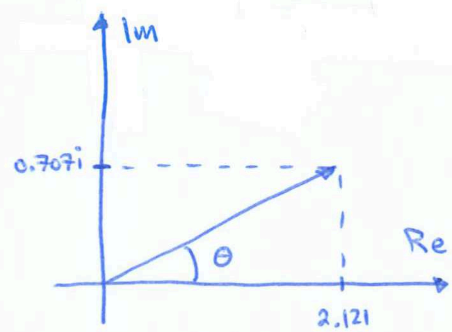
$$= 0.707 + 0.707i$$

$$= (2-i)(0.707 + 0.707i)$$

$$= 1.414 + 1.414i - 0.707i - 0.707i^2$$

$$= 0.707i + 1.414 + 0.707$$

$$= 2.121 + 0.707i$$



$$\theta = \tan^{-1}\left(\frac{0.707}{2.121}\right) = 0.322$$

$$R = \sqrt{2.121^2 + 0.707^2} = 2.24$$

$$\boxed{\begin{aligned} Z &= 2.121 + 0.707i \\ &= 2.24\, e^{0.322i} \end{aligned}}$$

(6 marks) Solve for z in either rectangular or polar form:

$$\frac{3z+2}{1+zi}+2=0$$

$$3z+2=-2(1+zi)$$

$$3z+2=-2-2zi$$

$$3z+2zi=-4$$

$$z(3+2i)=-4$$

$$z=\frac{(-4)}{(3+2i)}\frac{(3-2i)}{(3-2i)}$$

$$z=\frac{-12+8i}{9-6i+6i-4i^2}$$

$$z=\frac{-12+8i}{13}$$

$$\boxed{z=-\frac{12}{13}+\frac{8}{13}i}$$

$$z^3=27$$

$$z=(27)^{\frac{1}{3}}$$

$\theta=0$ $$z=\left(27e^{i0}\right)^{\frac{1}{3}}=27^{\frac{1}{3}}e^{\frac{0}{3}i}=3e^{0i}=3+0i$$

$\theta=2\pi$ $$z=\left(27e^{2\pi i}\right)^{\frac{1}{3}}=27^{\frac{1}{3}}e^{\frac{2\pi}{3}i}=3e^{\frac{2\pi}{3}i}=-1.5+2.6i$$

$\theta=4\pi$ $$z=\left(27e^{4\pi i}\right)^{\frac{1}{3}}=27^{\frac{1}{3}}e^{\frac{4\pi}{3}i}=3e^{\frac{4\pi}{3}i}=-1.5-2.6i$$

$$z + \frac{1}{z+1} = 0$$

$$z(z+1) + 1 = 0(z+1)$$

$$z^2 + z + 1 = 0$$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1^2 - 4}}{2}$$

$$= -\frac{1}{2} \pm \frac{\sqrt{-3}}{2}$$

$$= -0.5 \pm 0.87i$$

$$\boxed{z = -0.5 \pm 0.87i}$$

(6 marks) Determine the derivative with respect to x of the following equations.

$$y = \frac{4x^3}{7} + 2x^{\frac{3}{4}}$$

$$\boxed{\frac{dy}{dx} = \frac{12x^2}{7} + \frac{3}{2}x^{-\frac{1}{4}}}$$

$$y = x^2 \sin(x^{a+1} - bx + c) - \frac{1}{x}$$

$$\boxed{\frac{dy}{dx} = 2x \sin\left(x^{a+1} - bx + c\right) + x^2 \cos\left(x^{a+1} - bx + c\right)\left((a+1)x^a - b\right) + \frac{1}{x^2}}$$

$$y^2x - x = y\sin(x) + 2$$

$$\boxed{2y\frac{dy}{dx}x + y^2 - 1 = \frac{dy}{dx}\sin(x) + y\cos(x)}$$