Instructor:	Frank Secretain
Course:	Math 7
Date:	November 27, 2025
Assessment:	Test 3
Time allowed:	110 minutes
Devices allowed:	Pencil, pen, eraser, calculator
Notes from instructor:	Be neat. Show your work where needed. Box final answers.
Marks allocated:	4 question worth 20 marks
Percentage of final grade:	20% of final grade

DERIVATIVE RULES

Constant rule	$\frac{d}{dx}(C) = 0$
Multiplier rule	$\frac{d}{dx}\left(C \cdot f(x)\right) = C \cdot f'(x)$
Sum/difference rule	$\frac{d}{dx}\left(f(x) \pm g(x)\right) = f'(x) \pm g'(x)$
Product rule	$\frac{d}{dx}\left(f(x) \cdot g(x)\right) = f'(x)g(x) + f(x)g'(x)$
Quotient rule	$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - g'(x)f(x)}{\left(g(x)\right)^2}$
Chain rule	$\frac{d}{dx}\left(f\left(g(x)\right)\right) = f'\left(g(x)\right) \cdot g'(x)$
Power rule	$\frac{d}{dx}(x^n) = nx^{n-1}$
Exponent rule	$\frac{d}{dx}(e^x) = e^x$ $\quad$ $\frac{d}{dx}(b^x) = b^x \cdot \ln b$
Logarithm rule	$\frac{d}{dx}(\ln x) = \frac{1}{x}$ $\quad$ $\frac{d}{dx}(\log_b x) = \frac{1}{x} \cdot \frac{1}{\ln b}$
Trigonometric rules	$\frac{d}{dx}(\sin x) = \cos x$ $\quad$ $\frac{d}{dx}(\cos x) = -\sin x$

Formula Sheet

Order of Operations

$$ac + bc = c(a+b)$$

exponent rules

$$a^n a^m = a^{n+m} \qquad (a^n)^m = a^{nm} \qquad (ab)^n = a^n b^n$$

$$a^0 = 1 \qquad a^{-n} = \frac{1}{a^n} \qquad a^{\frac{n}{m}} = \sqrt[m]{a^n}$$

logarithmic rules

$$\log_b(b^x) = x \qquad b^{\log_b a} = a \qquad \log_b x = \frac{\log(x)}{\log(b)}$$

$$\log(xy) = \log(x) + \log(y) \qquad \log(x/y) = \log(x) - \log(y)$$

Forms of a 1st order polynomial $y = ax + b$

Forms of a 2nd order polynomial

$$y = ax^2 + bx + c$$ (expanded form)

$$y = a(x-h)^2 + k$$ (vertex form)

$$y = a(x-m)(x-n)$$ (factored form)

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Definition of the derivative

$$\frac{d}{dx} f(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Euler's formula

$$e^{i\theta} = \cos\theta + i \sin\theta$$

Rules of differentiation

$$\frac{\mathrm{d}}{\mathrm{dx}}(fg) = \frac{\mathrm{d}(f)}{\mathrm{dx}} g + f \frac{\mathrm{d}(g)}{\mathrm{dx}}$$

(product rule)

$$\frac{\mathrm{d}}{\mathrm{dx}}\left(\frac{f}{g}\right) = \frac{\frac{\mathrm{d}(f)}{\mathrm{dx}} g - f \frac{\mathrm{d}(g)}{\mathrm{dx}}}{g^2}$$

(quotient rule)

$$\frac{\mathrm{d}}{\mathrm{dx}}(f(g)) = \frac{\mathrm{d}(f)}{\mathrm{dx}} \frac{\mathrm{d}(g)}{\mathrm{dx}}$$

(chain rule)

Derivatives of select functions

$$\frac{d}{dx}(ax^n) = anx^{n-1}$$

$$\frac{d}{dx}(sin(x)) = cos(x)$$

$$\frac{d}{dx}(cos(x)) = -sin(x)$$

$$\frac{d}{dx}(tan(x)) = \frac{1}{(cos(x))^2}$$

$$\frac{d}{dx}(a^x) = a^x ln(a)$$

$$\frac{d}{dx}(log_a x) = \frac{1}{xln(a)}$$

Integrals of select functions

$$\int ax^n dx = \left\{ \begin{array}{ll} \frac{a}{n+1} x^{n+1} & , n \neq -1 \\ ln(|x|) & , n = -1 \end{array} \right\}$$ (polynomials)

$$\int sin(ax)dx = -\frac{1}{a} cos(ax)$$

$$\int cos(ax)dx = \frac{1}{a} sin(ax)$$ (trigonometry)

$$\int tan(ax)dx = ln(|sec(x)|)$$

$$\int a^x dx = \frac{1}{ln(a)} a^x$$

$$\int ln(x)dx = xln(x) - x$$ (exponentials)

(5 marks) Evaluate to either rectangular or polar form:

$$z = i + i(2 + i) + 2$$

$$z = (2 + i - e^{\frac{\pi}{2} i})^3$$

$$z = \frac{e^{2i}}{2-i}$$

(5 marks) Solve for z in either rectangular or polar form:

$$3z - iz + 3 = 1$$

$$z(z^2 - 27) + 2(i - 1) + 2 = 2i$$

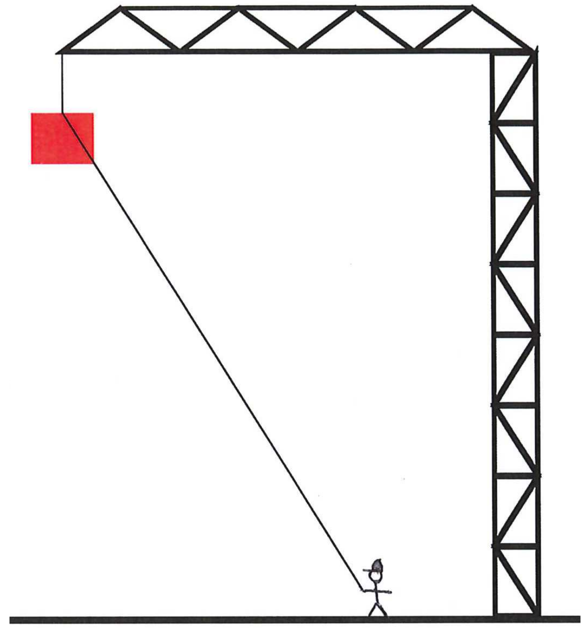
$$z + \frac{1}{z+1} = 0$$

(5 marks) Determine the derivative with respect to x of the following equations.

$$y = x^2 \sin(2ax^2 + b^2) + 3$$

$$y = x^2 y^2 - xy$$

(5 marks) A construction crane is lifting a load straight up from the ground while a worker is standing 30 feet away from the point directly beneath the load. The crane is lifting the load at a rate of 5 feet per second. How fast does the worker have to release the rope attached to the load when the load is 40 feet above the ground?

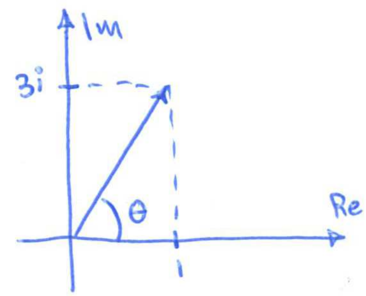


(5 marks) Evaluate to either rectangular or polar form:

$$z = i + i(2+i) + 2$$

$= i + 2i + i^2 + 2$

$= 1 + 3i$



$\theta = \tan^{-1}\left(\frac{3}{1}\right) = 1.249$

$R = \sqrt{3^2 + 1^2} = \sqrt{10}$

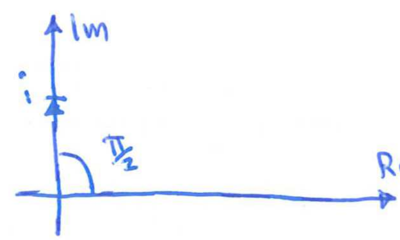
$$\boxed{Z = 1 + 3i = \sqrt{10}\, e^{1.25i} = 3.16\, e^{1.25i}}$$

$$z = (2 + i - e^{\frac{\pi}{2}i})^3$$

$= (2 + i - i)^3$

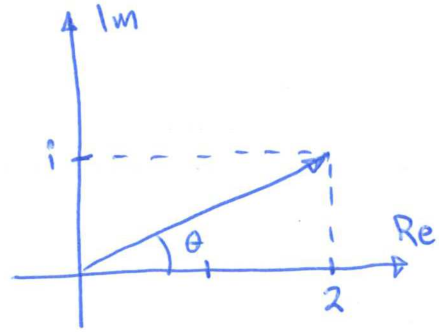
$= (2)^3$

$= 8$



$$\boxed{Z = 8 + 0i = 8e^{i0}}$$

$$z = \frac{e^{2i}}{2-i}$$



$$\theta = \tan^{-1}\left(\frac{1}{2}\right) = 0.4636$$

$$R = \sqrt{2^2 + 1^2} = \sqrt{5}$$

$$= \frac{e^{2i}\,(2+i)}{(2-i)(2+i)}$$

$$= \frac{(2+i)e^{2i}}{4+2i-2i-i^2}$$

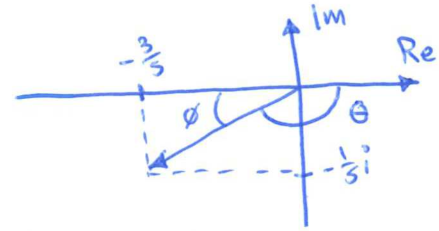
$$= \frac{\left(\sqrt{5}\,e^{0.4636i}\right)\left(e^{2i}\right)}{5}$$

$$= \frac{\sqrt{5}\,e^{(2+0.4636)i}}{5}$$

$$= \frac{\sqrt{5}}{5}e^{2.4636i} = \frac{\sqrt{5}}{5}\left(\cos(2.4636) + i\sin(2.4636)\right)$$

$$\boxed{\begin{aligned} z = -0.3483 + 0.2805i &= \frac{\sqrt{5}}{5}e^{2.4636i} \\ &= 0.447\,e^{2.4636i} \end{aligned}}$$

(5 marks) Solve for z in either rectangular or polar form:



$$3z - iz + 3 = 1$$

$z(3-i) = -2$

$z = \frac{-2}{(3-i)}\frac{(3+i)}{(3+i)}$

$= \frac{-6-2i}{9+3i-3i-i^2}$

$= -\frac{6}{10} - \frac{2}{10}i$

$\phi = \tan^{-1}\left(\frac{1}{5} / \frac{3}{5}\right) = 0.321$

$\theta = \pi - 0.321 = 2.820$

$R = \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{1}{5}\right)^2} = \frac{\sqrt{10}}{5}$

$$\boxed{z = -\frac{3}{5} - \frac{1}{5}i = \frac{\sqrt{10}}{5}e^{-2.82i} = 0.632\,e^{-2.82i}}$$

$$z(z^2 - 27) + 2(i-1) + 2 = 2i$$

$z^3 - 27z + \cancel{2i} - \cancel{2} + \cancel{2} = \cancel{2i}$

$z(z^2 - 27) = 0$

$z^2 = 27$

$z = (27)^{\frac{1}{2}}$

$\theta = 0$ $\quad = (27e^{i0})^{\frac{1}{2}} = \sqrt{27}$

$$\boxed{z = 0, \sqrt{27}, -\sqrt{27}}$$

$\theta = 0 + 2\pi$ $\quad z = (27e^{2\pi i})^{\frac{1}{2}} = -\sqrt{27}$

$$z + \frac{1}{z+1} = 0$$

$$z(z+1) + 1 = 0(z+1)$$

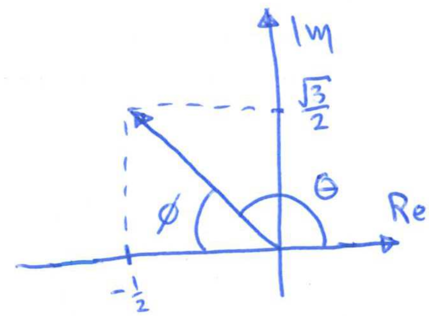
$$z^2 + z + 1 = 0$$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{(1)^2 - 4}}{2}$$

$$= \frac{-1}{2} \pm \frac{\sqrt{-3}}{2}$$

$$\boxed{z = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i = e^{\pm 2.094 i}}$$



$$\phi = \tan^{-1}\left(\frac{\sqrt{3}/2}{1/2}\right) = 1.047$$

$$\theta = \pi - 1.047 = 2.094$$

$$R = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{2}{2} = 1$$

(5 marks) Determine the derivative with respect to x of the following equations.

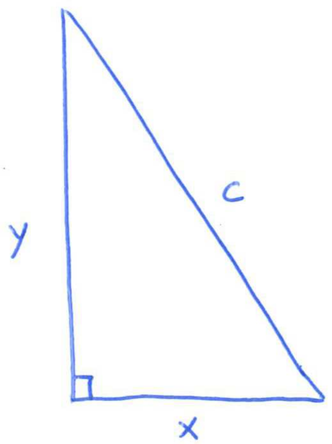
$$y = x^2 \sin(2ax^2 + b^2) + 3$$

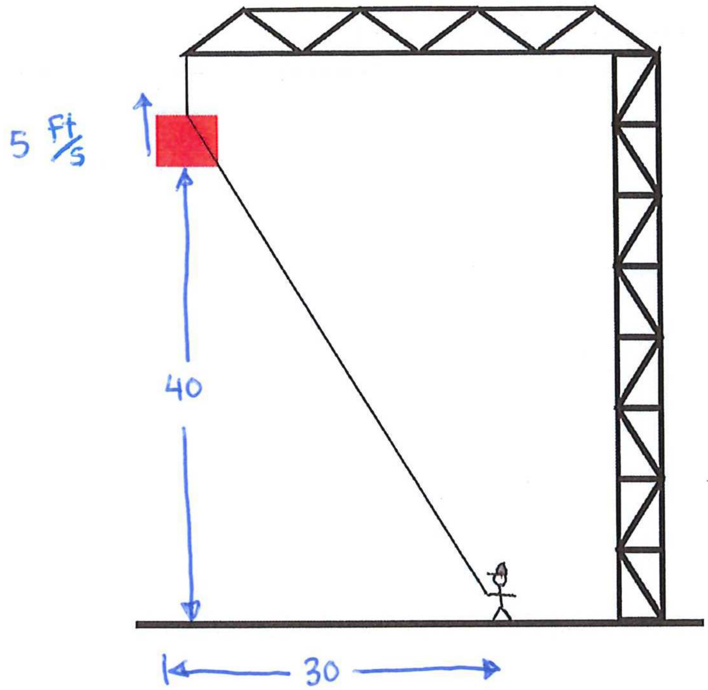
$$\boxed{\frac{dy}{dx} = 2x \sin\left(2ax^2 + b^2\right) + x^2 \cos\left(2ax^2 + b^2\right)\left(4ax\right)}$$

$$y = x^2y^2 - xy$$

$$\boxed{\frac{dy}{dx} = 2xy^2 + 2x^2y\frac{dy}{dx} - y - x\frac{dy}{dx}}$$

(5 marks) A construction crane is lifting a load straight up from the ground while a worker is standing 30 feet away from the point directly beneath the load. The crane is lifting the load at a rate of 5 feet per second. How fast does the worker have to release the rope attached to the load when the load is 40 feet above the ground?





$$x^2 + y^2 = c^2$$

$$\frac{d}{dt}\Big(\qquad \overset{0\ (\text{constant})}{2x\cancel{\frac{dx}{dt}}} + 2y\frac{dy}{dt} = 2c\frac{dc}{dt}$$

$$\frac{dc}{dt} = \frac{y}{c}\frac{dc}{dt}$$

$$= \frac{40}{50}(5)$$

$$= 4$$

@ $x = 30$, $y = 40$

$$c = \sqrt{x^2 + y^2}$$

$$= \sqrt{(30)^2 + (40)^2}$$

$$= 50$$

$$\boxed{\frac{dc}{dt} = 4\ \text{Ft}/\text{s}}$$

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Exponent rule	$\frac{d}{dx}(e^x) = e^x$ $\frac{d}{dx}(b^x) = b^x \cdot \ln b$
Logarithm rule	$\frac{d}{dx}(\ln x) = \frac{1}{x}$ $\frac{d}{dx}(\log_b x) = \frac{1}{x} \cdot \frac{1}{\ln b}$
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Formula Sheet

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$$ac + bc = c(a+b)$$

exponent rules

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$$a^0 = 1 \qquad a^{-n} = \frac{1}{a^n} \qquad a^{\frac{n}{m}} = \sqrt[m]{a^n}$$

logarithmic rules

$$\log_b(b^x) = x \qquad b^{\log_b a} = a \qquad \log_b x = \frac{\log(x)}{\log(b)}$$

$$\log(xy) = \log(x) + \log(y) \qquad \log(x/y) = \log(x) - \log(y)$$

Forms of a 1st order polynomial

$$y = ax + b$$

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$$\frac{\mathrm{d}}{\mathrm{dx}}(fg) = \frac{\mathrm{d}(f)}{\mathrm{dx}} g + f \frac{\mathrm{d}(g)}{\mathrm{dx}}$$

(product rule)

$$\frac{\mathrm{d}}{\mathrm{dx}}\left(\frac{f}{g}\right) = \frac{\frac{\mathrm{d}(f)}{\mathrm{dx}} g - f \frac{\mathrm{d}(g)}{\mathrm{dx}}}{g^2}$$

(quotient rule)

$$\frac{\mathrm{d}}{\mathrm{dx}}(f(g)) = \frac{\mathrm{d}(f)}{\mathrm{dx}} \frac{\mathrm{d}(g)}{\mathrm{dx}}$$

(chain rule)

Derivatives of select functions

$$\frac{d}{dx}(ax^n) = anx^{n-1}$$

$$\frac{d}{dx}(sin(x)) = cos(x)$$

$$\frac{d}{dx}(cos(x)) = -sin(x)$$

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$$\frac{d}{dx}(a^x) = a^x ln(a)$$

$$\frac{d}{dx}(log_a x) = \frac{1}{x ln(a)}$$

Integrals of select functions

$$\int ax^n dx = \left\{ \begin{array}{ll} \frac{a}{n+1} x^{n+1} & , n \neq -1 \\ ln(|x|) & , n = -1 \end{array} \right\}$$ (polynomials)

$$\int sin(ax) dx = -\frac{1}{a} cos(ax)$$

$$\int cos(ax) dx = \frac{1}{a} sin(ax)$$ (trigonometry)

$$\int tan(ax) dx = ln(|sec(x)|)$$

$$\int a^x dx = \frac{1}{ln(a)} a^x$$ (exponentials)

$$\int ln(x) dx = x ln(x) - x$$

(5 marks) Evaluate to either rectangular or polar form:

$$z = 3i - 2(1 + i)$$

$$z = (2 - i)^3$$

$$z = (1 - i)e^{\frac{3i}{2}}$$

(5 marks) Solve for z in either rectangular or polar form:

$$\frac{z-i}{2-i}+2=0$$

$$z^3+1+i=9+i$$

$$\frac{1}{z} + 2z = 1$$

(5 marks) Determine the derivative with respect to x of the following equations.

$$y = x\sin(x^2 - 5) + 2$$

$$y = xy^2 - sin(y) + 3x$$

(5 Marks) Suppose that we have two resistors connected in parallel with resistances R1 and R2. The total resistance, R, is then given by:

$$\frac{1}{R} = \frac{1}{R1} + \frac{1}{R2}$$

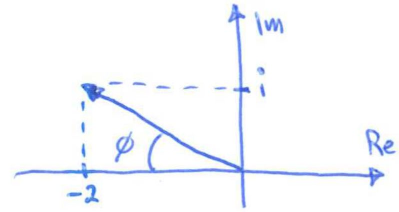
Suppose that R1 is increasing at a rate of 0.4 ohms/min and R2 is decreasing at a rate of 0.7 ohms/min. At what rate is R changing when R1=80 ohms and R2=100 ohms?

(5 marks) Evaluate to either rectangular or polar form:

$$z = 3i - 2(1+i)$$

$$= 3i - 2 - 2i$$

$$= -2 + i$$



$\phi = \tan^{-1}\left(\frac{1}{2}\right) = 0.464$

$\theta = \pi - 0.464 = 2.678$

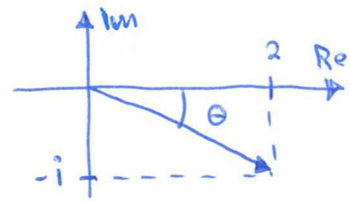
$R = \sqrt{(-2)^2 + 1^2} = \sqrt{5}$

$$\boxed{z = -2 + i = \sqrt{5}\, e^{2.678i} = 2.24\, e^{2.678i}}$$

$$z = (2-i)^3$$

$$= \left(\sqrt{5}\, e^{-0.464i}\right)^3$$

$$= \left(\sqrt{5}\right)^3 e^{-1.391i}$$



$\theta = \tan^{-1}\left(\frac{1}{2}\right) = 0.464$

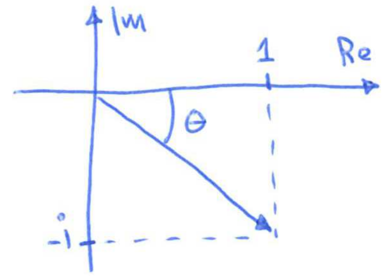
$R = \sqrt{2^2 + (-1)^2} = \sqrt{5}$

$$\boxed{z = 5^{\frac{3}{2}} e^{-1.391i} = 11.18\, e^{-1.391i} = 2 - 11i}$$

$$z = (1-i)e^{\frac{3i}{2}}$$

$$= \left(\sqrt{2}\, e^{-\frac{\pi i}{4}}\right) e^{\frac{3i}{2}}$$

$$= \sqrt{2}\, e^{\left(\frac{3}{2} - \frac{\pi}{4}\right) i}$$



$\theta = \frac{\pi}{4}$

$R = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

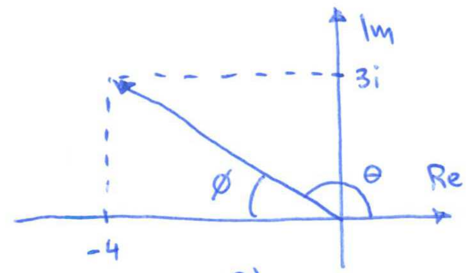
$$Z = \sqrt{2}\, e^{\left(\frac{3}{2} - \frac{\pi}{4}\right) i}$$

$$= 1.414\, e^{0.715 i}$$

$$= 1.068 + 0.927 i$$

(5 marks) Solve for z in either rectangular or polar form:

$$\frac{z-i}{2-i}+2=0$$



$\phi = \tan^{-1}\left(\frac{3}{4}\right) = 0.644$

$\theta = \pi - \phi = 2.5$

$R = \sqrt{(-4)^2 + (3)^2} = 5$

$$z - i = -2(2-i)$$

$$z = -4 + 2i + i$$

$$\boxed{\begin{aligned} z &= -4 + 3i \\ &= 5e^{2.5i} \end{aligned}}$$

$$z^3 + 1 + i = 9 + i$$

$$z^3 = 8$$

$$z = (8)^{\frac{1}{3}}$$

$\theta = 0$ $\quad = \left(8e^{0i}\right)^{\frac{1}{3}} = 2e^{i0} = 2 + 0i$

$\theta = 0 + 2\pi$ $\quad = \left(8e^{2\pi i}\right)^{\frac{1}{3}} = 2e^{\frac{2\pi}{3}i} = -1 + 1.732i$

$\theta = 0 + 4\pi$ $\quad = \left(8e^{4\pi i}\right)^{\frac{1}{3}} = 2e^{\frac{4\pi}{3}i} = -1 - 1.732i$

$$\frac{1}{z} + 2z = 1$$

$$1 + 2z^2 = z$$

$$2z^2 - z + 1 = 0$$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{1 \pm \sqrt{1-8}}{4}$$

$$= \frac{1}{4} \pm \frac{1}{4}\sqrt{-7}$$

$$= \frac{1}{4} \pm \frac{\sqrt{7}}{4} i$$

$$\boxed{\begin{aligned} z &= \frac{1}{4} \pm \frac{\sqrt{7}}{4} i \\ &= 0.25 \pm 0.66 i \end{aligned}}$$

(5 marks) Determine the derivative with respect to x of the following equations.

$$y = x\sin(x^2 - 5) + 2$$

$\frac{d}{dx}$

$$\boxed{\frac{dy}{dx} = \sin(x^2-5) + x\cos(x^2-5)(2x)}$$

$$y = xy^2 - sin(y) + 3x$$

$\frac{d}{dx}$

$$\frac{dy}{dx} = y^2 + 2xy\frac{dy}{dx} - \cos(y)\frac{dy}{dx} + 3$$

$$\boxed{\frac{dy}{dx} = \frac{y^2 + 3}{1 - 2xy + \cos(y)}}$$

(5 Marks) Suppose that we have two resistors connected in parallel with resistances R1 and R2. The total resistance, R, is then given by:

$$\frac{1}{R} = \frac{1}{R1} + \frac{1}{R2}$$

Suppose that R1 is increasing at a rate of 0.4 ohms/min and R2 is decreasing at a rate of 0.7 ohms/min. At what rate is R changing when R1=80 ohms and R2=100 ohms?

$$R^{-1} = R_1^{-1} + R_2^{-1}$$

$\frac{d}{dt}$

$$-R^{-2} \cdot \frac{dR}{dt} = -R_1^{-2}\frac{dR_1}{dt} - R_2^{-2}\frac{dR_2}{dt}$$

$$\frac{1}{R^2}\frac{dR}{dt} = \frac{1}{R_1^2}\frac{dR_1}{dt} + \frac{1}{R_2^2}\frac{dR_2}{dt}$$

$$\frac{dR}{dt} = R^2\left(\frac{1}{R_1^2}\frac{dR_1}{dt} + \frac{1}{R_2^2}\frac{dR_2}{dt}\right)$$

$$= (44.4)^2\left(\frac{1}{80^2}(+0.4) + \frac{1}{100^2}(-0.7)\right)$$

$$= -0.0148 \text{ ohms/min}$$

$$R = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

$$= \left(\frac{1}{80} + \frac{1}{100}\right)^{-1}$$

$$= 44.4 \text{ ohms}$$

$$\boxed{\frac{dR}{dt} = -0.015 \text{ ohms/min}}$$