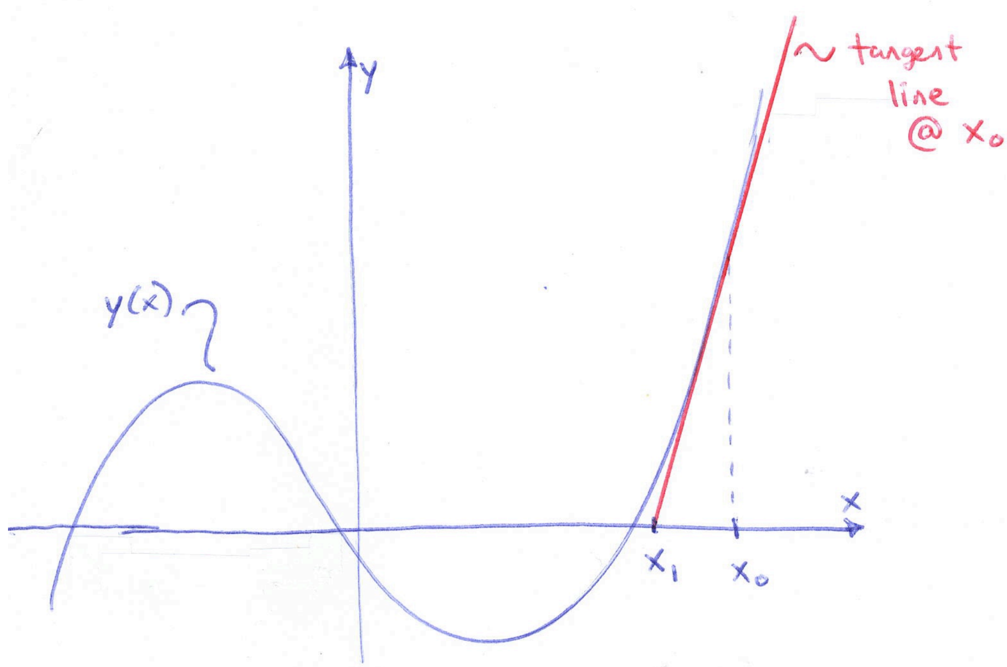
Root Finding Project

This project is designed to find roots (i.e. where a function equals zero) using numerical approximation methods as applied to this course. In this project, the intercepts of a circle and a parabola will be calculated using Newton's method. Newton's method is a step by step (iterative) numerical solution (using a computer) of the problem, as apposed to a analytical solution which we have been doing in class.



Groups of 1-2 people

Report due week 14
Worth 10% of final grade
Report out of 10 marks

Outline:

This report will outline the project by means of an example. It should only be used as a template and you should create your own report and excel file for your submission. In this example the intercepts of a ellipse and parabola will be calculated.

Instructions:

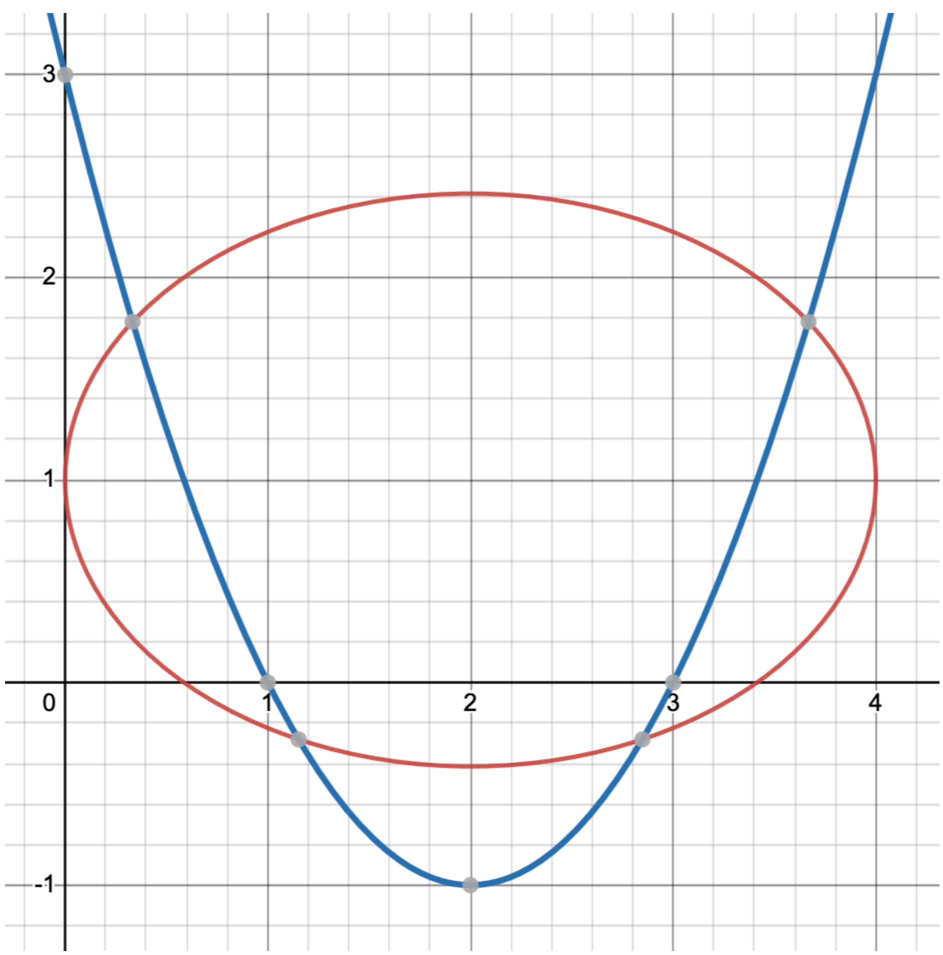
State the two functions you will find the intercepts.

Below is the ellipse and parabola that the intercepts will be calculated.

$$\frac{(y-1)^2}{2} + \frac{(x-2)^2}{4} = 1 \qquad (1)$$

$$y + 1 = (x-2)^2 \qquad (2)$$

Below is the graph of the two functions stated above.



(3 marks) Derive and develop an excel routine that will solve.

Below is the substitution of the parabola into the ellipse.

solve for y in (2)

$$y = (x-2)^2 - 1$$
$$= (x-2)(x-2) - 1$$
$$= (x-2)x - (x-2)2 - 1$$
$$= x^2 - 2x - 2x + 4 - 1$$
$$y = x^2 - 4x + 3 \qquad (2a)$$

sub (2a) into (1)

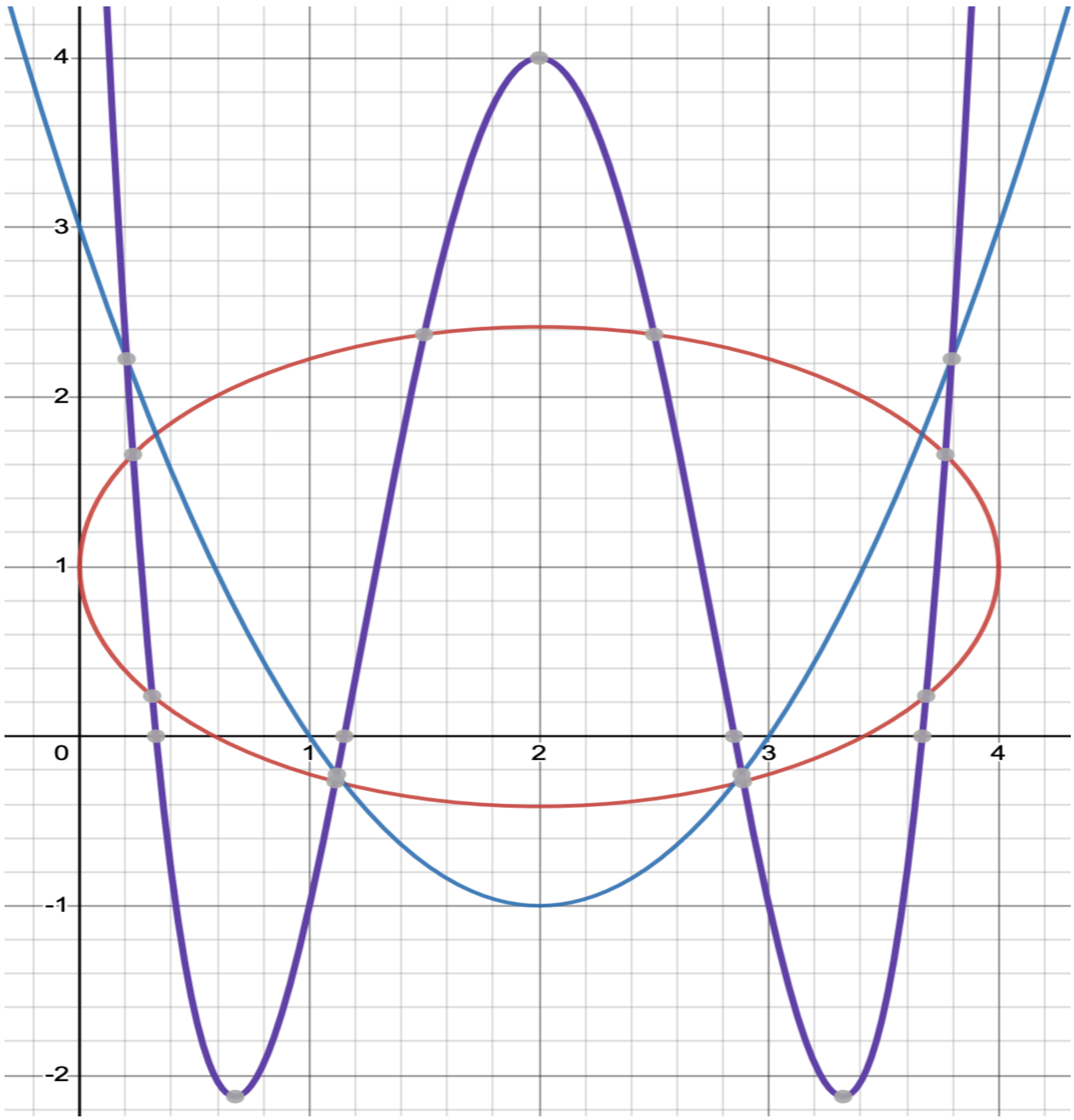
$$\frac{([x^2-4x+3]-1)^2}{2} + \frac{(x-2)^2}{4} = 1$$
$$2(x^2-4x+2)^2 + (x-2)^2 = 4$$
$$2(x^2-4x+2)(x^2-4x+2) + (x^2-4x+4) = 4$$
$$2\left[(x^2-4x+2)x^2 - (x^2-4x+2)4x + (x^2-4x+2)2\right] + x^2 - 4x + 4 = 4$$
$$2\left[x^4 - 4x^3 + 2x^2 - 4x^3 + 16x^2 - 8x + 2x^2 - 8x + 4\right] + x^2 - 4x = 0$$
$$2\left[x^4 - 8x^3 + 20x^2 - 16x + 4\right] + x^2 - 4x = 0$$
$$2x^4 - 16x^3 + 40x^2 - 32x + 8 + x^2 - 4x = 0$$
$$\boxed{2x^4 - 16x^3 + 41x^2 - 36x + 8 = 0} \qquad (1a)$$

this leads to a 4th order polynomial equalling zero.

Lets let the 4th order polynomial equal y to create a function. Therefore, we will need to find the roots of that function (see below).

let

$$y = 2x^4 - 16x^3 + 41x^2 - 36x + 8$$



The purple curve is the 4th order polynomial and you can see that the roots of the polynomial line up with the intercepts of the parabola and ellipse.

We will use Newton's method to determine the tangent line at a given point and solve for it's root and re-iterate, as shown below.

let

$$y = 2x^4 - 16x^3 + 41x^2 - 36x + 8$$

so

$$\frac{dy}{dx} = 8x^3 - 48x^2 + 82x - 36$$

and

$$y_o = 2x_o^4 - 16x_o^3 + 41x_o^2 - 36x_o + 8$$

and

$$\left.\frac{dy}{dx}\right|_{x_o} = 8x_o^3 - 48x_o^2 + 82x_o - 36$$

$$= \text{slope} = a$$

so the tangent line is:

$$y = ax + b \qquad \text{where} \qquad a = \left.\frac{dy}{dx}\right|_{x_o}$$

rearrange for b and substitute (x_o, y_o)

$$b = y_o - ax_o$$

determine x_1 point by setting $y = 0$

$$\boxed{x_1 = -\frac{b}{a}}$$

Now we have a procedure that can be implemented into excel to iteratively solve for a root given an initial guess (x_o).

(3 marks) Implement into excel.

Below is the implementation of the above derivation into excel for x_o = 0.

xo	yo	dy/dx|xo	b	x1
0.00	**8.00**	**-36.00**	**8.00**	**0.22**

Below we continue the iterative approach by letting x_o equal the previously determined x_1 value and reevaluate.

xo	yo	dy/dx|xo	b	x1
0.00	**8.00**	**-36.00**	**8.00**	**0.22**
0.22	1.85	-20.06	6.31	0.31
0.31	0.25	-14.70	4.88	0.33
0.33	0.01	-13.78	4.58	0.33
0.33	0.00	-13.75	4.57	0.33
0.33	0.00	-13.75	4.57	0.33
0.33	0.00	-13.75	4.57	0.33
0.33	0.00	-13.75	4.57	0.33
0.33	0.00	-13.75	4.57	0.33
0.33	0.00	-13.75	4.57	0.33
0.33	0.00	-13.75	4.57	0.33
0.33	0.00	-13.75	4.57	0.33
0.33	0.00	-13.75	4.57	0.33
0.33	0.00	-13.75	4.57	0.33
0.33	0.00	-13.75	4.57	0.33
0.33	0.00	-13.75	4.57	0.33
0.33	0.00	-13.75	4.57	0.33
0.33	0.00	-13.75	4.57	0.3324339873923
0.33	0.00	-13.75	4.57	0.3324339873923

This ends with the root being x=0.3324229873

We will repeat this process for other guesses close to the root and determine all the root being:

x =	0.332433987392
x =	1.151929487840
x =	2.848070512160
x =	3.667566012608

(1 mark) Enter the biggest root into blackboard.

Marking Scheme:

Write up a report (such as this project outline) that includes:

a. (2 marks) the derivation of Newton's method applied to your function.
b. (2 marks) plot of original functions and the 4th order polynomial function.
c. (3 mark) biggest root inputted into blackboard.
d. (3 marks) neatly laid out and formatted document.
e. signed workload distribution (Appendix A).

Appendix A

Workload Distribution

Name	Signature	Percentage of load	Contribution to Project