Integration by Substitution:

$$\int -16x^3(-4x^4-1)^{-5}\,dx$$

let $u = -4x^4 - 1$

$\frac{d}{dx}\Big($ $\frac{du}{dx} = -16x^3 \Rightarrow dx = \frac{du}{-16x^3}$

$$\int \cancel{-16x^3}(u)^{-5}\left(\frac{du}{\cancel{-16x^3}}\right)$$

$$\int u^{-5}\,du$$

$$\frac{1}{-4}u^{-4} + C$$

$$\frac{1}{-4}(-4x^4-1)^{-4} + C$$

$$\boxed{\int -16x^3(-4x^4-1)^{-5}\,dx = -\frac{1}{4}(-4x^4-1)^{-4} + C}$$

$$\int -\frac{8x^3}{(-2x^4+5)^5}\,dx$$

let $u = -2x^4 + 5$

$$\frac{du}{dx} = -8x^3 \quad \Rightarrow \quad dx = \frac{du}{-8x^3}$$

$$\int -\frac{\cancel{8x^3}}{(u)^5}\left(\frac{du}{-\cancel{8x^3}}\right)$$

$$\int u^{-5}\,du$$

$$-\frac{1}{4}u^{-4} + C$$

$$-\frac{1}{4}(-2x^4+5)^{-4} + C$$

$$\int -\frac{8x^3}{(-2x^4+5)^5}\,dx = -\frac{1}{4}(-2x^4+5)^{-4} + C$$

$$\int (3x^5 - 3)^{3/5}\, 15x^4\, dx$$

let $u = 3x^5 - 3$

$$\frac{du}{dx} = 15x^4 \Rightarrow dx = \frac{du}{15x^4}$$

$$\int (u)^{3/5}\, \cancel{15x^4}\left(\frac{du}{\cancel{15x^4}}\right)$$

$$\int u^{3/5}\, du$$

$$\frac{5}{8}u^{8/5} + C$$

$$\frac{5}{8}(3x^5 - 3)^{8/5} + C$$

$$\boxed{\int (3x^5 - 3)^{3/5}\, 15x^4\, dx = \frac{5}{8}(3x^5 - 3)^{8/5} + C}$$

$$\int (e^{4x} - 4)^{1/5}\, 8e^{4x}\, dx$$

let $u = e^{4x} - 4$

$$\frac{du}{dx} = 4e^{4x} \quad \Rightarrow \quad dx = \frac{du}{4e^{4x}}$$

$$\int (u)^{1/5}\, \cancel{8}^{\,2}\, \cancel{e^{4x}} \left(\frac{du}{\cancel{4e^{4x}}}\right)$$

$$\int 2u^{1/5}\, du$$

$$2\left(\frac{5}{6}u^{6/5}\right) + c$$

$$\frac{5}{3}(e^{4x} - 4)^{6/5} + C$$

$$\boxed{\int (e^{4x}-4)^{1/5}\, 8e^{4x}\, dx = \frac{5}{3}(e^{4x}-4)^{6/5} + c}$$

$$\int 5x\sqrt{2x+3}\;dx$$

let $u = 2x + 3$

$$\frac{du}{dx} = 2 \implies dx = \frac{du}{2}$$

$$\int 5x\left(u^{\frac{1}{2}}\right)\left(\frac{du}{2}\right)$$

remember that $u = 2x + 3$

$\implies x = \frac{u}{2} - \frac{3}{2}$

$$\int 5\left(\frac{u}{2} - \frac{3}{2}\right) u^{\frac{1}{2}} \left(\frac{du}{2}\right)$$

$$\int \frac{5}{4}u^{\frac{3}{2}} - \frac{15}{4}u^{\frac{1}{2}}\;du$$

$$\left(\frac{\cancel{5}}{4}\right)\left(\frac{2}{\cancel{5}}\right)u^{\frac{5}{2}} - \left(\frac{15}{4}\right)\left(\frac{2}{3}\right)u^{\frac{3}{2}} + C$$

$$\frac{1}{2}(2x+3)^{\frac{5}{2}} - \frac{5}{2}(2x+3)^{\frac{3}{2}} + C$$

$$\boxed{\int 5x(2x+3)^{\frac{1}{2}}dx = \frac{1}{2}(2x+3)^{\frac{5}{2}} - \frac{5}{2}(2x+3)^{\frac{3}{2}} + C}$$

$$\int \frac{2}{x(-1+\ln(4x))}\,dx$$

let

$$u = -1 + \ln(4x)$$

$$\frac{du}{dx} = \frac{1}{4x}(4) = \frac{1}{x} \Rightarrow dx = x\,du$$

$$\int \frac{2}{\cancel{x}(u)}(\cancel{x}\,du)$$

$$\int \frac{2}{u}\,du$$

$$2\ln|u| + c$$

$$2\ln|-1+\ln(4x)| + c$$

$$\boxed{\int \frac{2}{x(-1+\ln(4x))}\,dx = 2\ln|-1+\ln(4x)| + c}$$

$$\int \frac{5e^{(-3+\ln(3x))}}{x}\,dx$$

let $u = -3 + \ln(3x)$

$$\frac{du}{dx} = \frac{1}{3x}(3) = \frac{1}{x} \Rightarrow dx = x\,du$$

$$\int \frac{5e^u}{\cancel{x}}(\cancel{x}\,du)$$

$$\int 5e^u\,du$$

$$5e^u + C$$

$$5e^{(-3+\ln(3x))} + C$$

$$\boxed{\int \frac{5e^{(-3+\ln(3x))}}{x}\,dx = 5e^{(-3+\ln(3x))} + C}$$

$$\int 36x^2 e^{(4x^3+3)}\,dx$$

let $u = 4x^3 + 3$

$$\frac{du}{dx} = 12x^2 \Rightarrow dx = \frac{du}{12x^2}$$

$$\int 36\cancel{x^2} e^u \left(\frac{du}{12\cancel{x^2}}\right)$$

$$\int 3e^u\,du$$

$$3e^u + C$$

$$3e^{4x^3+3} + C$$

$$\boxed{\int 36x^2 e^{(4x^3+3)}\,dx = 3e^{(4x^3+3)} + C}$$

$$\int 10\sin(-2x)\, e^{\cos(-2x)}\, dx$$

let $u = \cos(-2x)$

$$\frac{du}{dx} = -\sin(-2x)(-2) = 2\sin(-2x)$$

$$dx = \frac{du}{2\sin(-2x)}$$

$$\int 10\,\cancel{\sin(-2x)}\, e^{u} \left(\frac{du}{2\,\cancel{\sin(-2x)}}\right)$$

$$\int 5e^{u}\, du$$

$$5e^{u} + C$$

$$5e^{\cos(-2x)} + C$$

$$\boxed{\int 10\sin(-2x)\, e^{\cos(-2x)}\, dx = 5e^{\cos(-2x)} + C}$$

$$\int -\frac{5\cos(-4+\ln(4x))}{x}\,dx$$

let

$$u = -4 + \ln(4x)$$

$$\frac{du}{dx} = \frac{1}{4x}(4) \Rightarrow dx = x\,du$$

$$\int -\frac{5\cos u}{\cancel{x}}\left(\cancel{x}\,du\right)$$

$$\int -5\cos u\,du$$

$$-5\sin u + C$$

$$-5\sin(-4+\ln(4x)) + C$$

$$\boxed{\int -\frac{5\cos(-4+\ln(4x))}{x}\,dx = -5\sin(-4+\ln(4x)) + C}$$

Integration by Parts

$$\int x e^{-x}\, dx$$

$u = x$ $\qquad$ $dv = e^{-x}\, dx$

$\frac{du}{dx} = 1$ $\qquad$ $\int dv = \int e^{-x}\, dx$

$du = dx$ $\qquad$ $v = -e^{-x}$

$$\int u\, dv = uv - \int v\, du$$

$$\int x e^{-x} dx = (x)(-e^{-x}) - \int -e^{-x}\, dx$$

$$= -x e^{-x} - e^{-x} + C$$

$$\boxed{\int x e^{-x}\, dx = -x e^{-x} - e^{-x} + C}$$

$$\int \sqrt{x}\,\ln(x)\,dx$$

$$u = \ln(x) \qquad dv = x^{\frac{1}{2}}\,dx$$

$$\frac{du}{dx} = \frac{1}{x} \qquad \int dv = \int x^{\frac{1}{2}}\,dx$$

$$du = \frac{dx}{x} \qquad v = \frac{2}{3}x^{\frac{3}{2}}$$

$$\int u\,dv = uv - \int v\,du$$

$$\int \sqrt{x}\,\ln(x)\,dx = (\ln(x))\left(\frac{2}{3}x^{\frac{3}{2}}\right) - \int \frac{2}{3}x^{\cancel{\frac{3}{2}}}\left(\frac{dx}{\cancel{x}}\right)$$

$$= \frac{2}{3}x^{\frac{3}{2}}\ln(x) - \int \frac{2}{3}x^{\frac{1}{2}}\,dx$$

$$= \frac{2}{3}x^{\frac{3}{2}}\ln(x) - \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)x^{\frac{3}{2}} + C$$

$$= \frac{2}{3}x^{\frac{3}{2}}\ln(x) - \frac{4}{9}x^{\frac{3}{2}} + C$$

$$\boxed{\int \sqrt{x}\,\ln(x)\,dx = \frac{2}{3}x^{\frac{3}{2}}\ln(x) - \frac{4}{9}x^{\frac{3}{2}} + C}$$

$$\int x^2 e^{5x}\,dx$$

$u = x^2 \qquad dv = e^{5x}\,dx$

$\frac{du}{dx} = 2x \qquad \int dv = \int e^{5x}\,dx$

$du = 2x\,dx \qquad v = \frac{1}{5}e^{5x}$

$$\int x^2 e^{5x} = (x^2)\left(\frac{1}{5}e^{5x}\right) - \int \frac{1}{5}e^{5x}(2x\,dx)$$

$$= \frac{x^2}{5}e^{5x} - \int \frac{2}{5}x e^{5x}\,dx$$

$w = x \qquad dz = e^{5x}\,dx$

$\frac{dw}{dx} = 1 \qquad \int dz = \int e^{5x}\,dx$

$dw = dx \qquad z = \frac{1}{5}e^{5x}$

$$\int x^2 e^{5x} = \frac{x^2}{5}e^{5x} - \frac{2}{5}\left[x\left(\frac{1}{5}e^{5x}\right) - \int \frac{1}{5}e^{5x}\,dx\right]$$

$$= \frac{x^2}{5}e^{5x} - \frac{2}{25}xe^{5x} + \left(\frac{2}{5}\right)\left(\frac{1}{5}\right)\left(\frac{1}{5}\right)e^{5x} + C$$

$$= \frac{x^2}{5}e^{5x} - \frac{2}{25}xe^{5x} + \frac{2}{125}e^{5x} + C$$

$$\boxed{\int x^2 e^{5x}\,dx = \frac{1}{5}x^2e^{5x} - \frac{2}{25}xe^{5x} + \frac{2}{125}e^{5x} + C}$$

$$\int \cos(2x)\, e^{-x}\, dx$$

$u = \cos(2x)$ $\quad$ $dv = e^{-x} dx$

$\frac{du}{dx} = -2\sin(2x)$ $\quad$ $\int dv = \int e^{-x} dx$

$du = -2\sin(2x)dx$ $\quad$ $v = -e^{-x}$

$$\int \cos(2x)\, e^{-x} dx = (\cos(2x))(-e^{-x}) - \int -e^{-x}(-2\sin(2x))\, dx$$

$$= -e^{-x}\cos(2x) - \int 2\sin(2x) e^{-x}\, dx$$

$w = 2\sin(2x)$ $\quad$ $dz = e^{-x} dx$

$\frac{dw}{dx} = 4\cos(2x)$ $\quad$ $\int dz = \int e^{-x} dx$

$dw = 4\cos(2x)dx$ $\quad$ $z = -e^{-x}$

$$\int \cos(2x)e^{-x} dx = -e^{-x}\cos(2x) - \left[2\sin(2x)(-e^{-x}) - \int -e^{-x}(4\cos(2x)\,dx)\right]$$

$$\int \cos(2x)e^{-x}dx = -e^{-x}\cos(2x) + 2e^{-x}\sin(2x) - 4\int \cos(2x)\, e^{-x}\, dx$$

$$5\int \cos(2x)e^{-x}dx = -e^{-x}\cos(2x) + 2e^{-x}\sin(2x)$$

$$\boxed{\int \cos(2x)e^{-x}dx = \frac{2}{5}e^{-x}\sin(2x) - \frac{1}{5}e^{-x}\cos(2x) + C}$$

$$\int \frac{x^2}{e^{2x}}\,dx$$

$u = x^2 \qquad dv = e^{-2x}dx$

$\frac{du}{dx} = 2x \qquad \int dv = \int e^{-2x}dx$

$du = 2x\,dx \qquad V = -\frac{1}{2}e^{-2x}$

$$\int \frac{x^2}{e^{2x}}\,dx = (x^2)\left(-\frac{1}{2}e^{-2x}\right) - \int -\frac{1}{\cancel{2}}e^{-2x}(\cancel{2}x\,dx)$$

$$= -\frac{x^2}{2}e^{-2x} + \int x e^{-2x}dx$$

$w = x \qquad dz = e^{-2x}dx$

$\frac{dw}{dx} = 1 \qquad \int dz = \int e^{-2x}dx$

$dw = dx \qquad z = -\frac{1}{2}e^{-2x}$

$$\int \frac{x^2}{e^{2x}}\,dx = -\frac{x^2}{2}e^{-2x} + \left[x\left(-\frac{1}{2}e^{-2x}\right) - \int -\frac{1}{2}e^{-2x}dx\right]$$

$$= -\frac{x^2}{2}e^{-2x} - \frac{x}{2}e^{-2x} + \frac{1}{2}\int e^{-2x}dx$$

$$\boxed{\int \frac{x^2}{e^{2x}}\,dx = -\frac{x^2}{2}e^{-2x} - \frac{x}{2}e^{-2x} - \frac{1}{4}e^{-2x} + C}$$

$$\int x \ln(x)\, dx$$

$u = x \qquad dv = \ln(x)\, dx$

$\frac{du}{dx} = 1 \qquad \int dv = \int \ln(x) dx$

$du = dx \qquad v = x\ln(x) - x$

$$\int x \ln(x)\, dx = (x)(x\ln(x) - x) - \int (x\ln(x) - x)\, dx$$

$$\int x \ln(x)\, dx = x^2\ln(x) - x^2 - \int x\ln(x)\, dx + \int x\, dx$$

$$2\int x \ln(x) dx = x^2\ln(x) - x^2 + \tfrac{1}{2}x^2 + C$$

$$2\int x \ln(x) dx = x^2\ln(x) - \frac{x^2}{2} + C$$

$$\boxed{\int x \ln(x) dx = \frac{x^2}{2}\ln(x) - \frac{x^2}{4} + C}$$

$$\int x^2 \ln(5x)\,dx$$

$u = x^2 \qquad dv = \ln(5x)\,dx$

$\frac{du}{dx} = 2x \qquad \int dv = \int \ln(5x)\,dx$

$du = 2x\,dx \qquad v = x\ln(5x) - x$

$$\int x^2 \ln(5x)\,dx = (x^2)(x\ln(5x) - x) - \int (x\ln(5x) - x)(2x\,dx)$$

$$\int x^2 \ln(5x)\,dx = x^3\ln(5x) - x^3 - \int 2x^2\ln(5x)\,dx + \int 2x^2\,dx$$

$$3\int x^2 \ln(5x)\,dx = x^3\ln(5x) - x^3 + \frac{2}{3}x^3 + C$$

$$3\int x^2 \ln(5x)\,dx = x^3\ln(5x) - \frac{1}{3}x^3 + C$$

$$\boxed{\int x^2 \ln(5x)\,dx = \frac{x^3}{3}\ln(5x) - \frac{x^3}{9} + C}$$