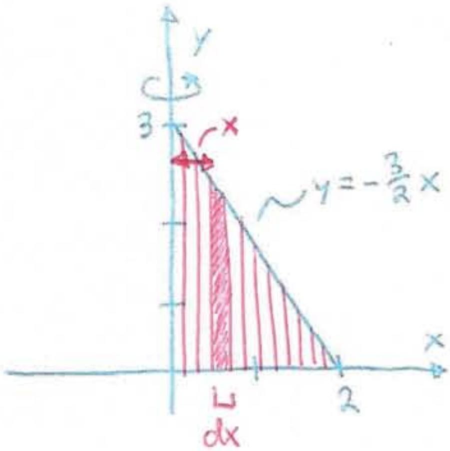
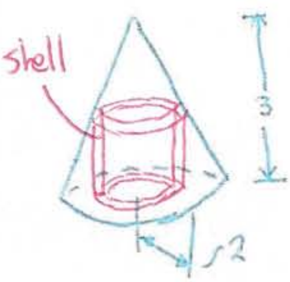
find the volume described in the figure



volume of revolved triangle?



use shells of height $y(x)$, radius of x and thickness dx.

$$\text{Volume} = \int_0^2 (2\pi r h)\,dx = \int_0^2 2\pi(x)(y(x))\,dx = \int_0^2 2\pi x\left(-\tfrac{3}{2}x + 3\right)dx$$

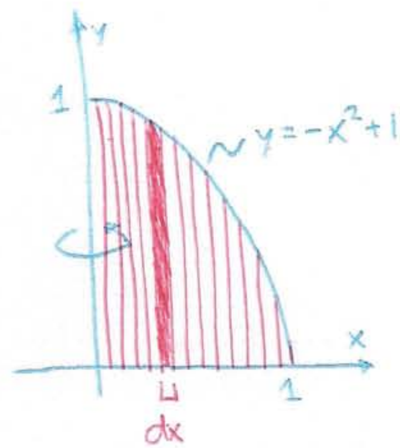
$$= \int_0^2 -3\pi x^2 + 6\pi x\,dx = \left[-\pi x^3 + 3\pi x^2\right]_0^2$$

$$= \left[-\pi(2)^3 + 3\pi(2)^2\right] - \left[-\pi(0)^3 + 3\pi(0)^2\right]$$

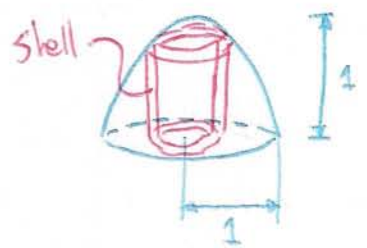
$$= -8\pi + 12\pi = 4\pi \cong 12.57 \text{ units}^3$$

$$\boxed{\text{Volume} = 4\pi \cong 12.57 \text{ units}^3}$$

find the volume described in the figure



Volume of enclosed parabola



use shells of height $y(x)$, radius of x
and thickness dx.

$$\text{Volume} = \int_0^1 (2\pi r h)\,dx = \int_0^1 2\pi(x)(y(x))\,dx = \int_0^1 2\pi x(-x^2+1)\,dx$$

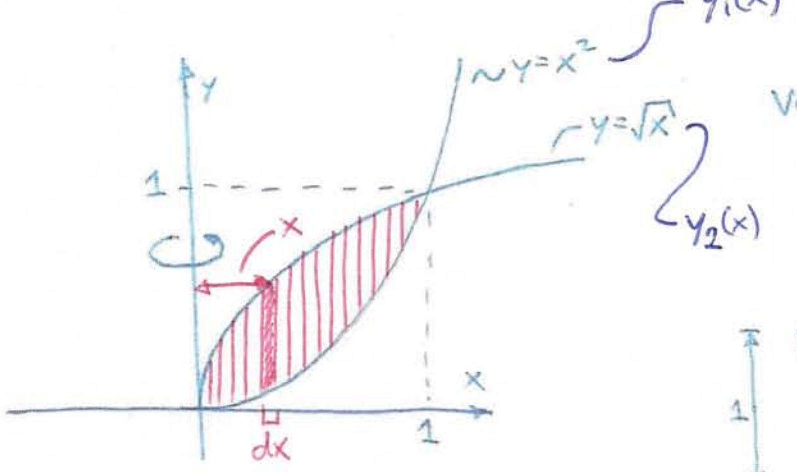
$$= \int_0^1 -2\pi x^3 + 2\pi x \; dx = \left[-\frac{2\pi}{4}x^4 + \frac{2\pi}{2}x^2\right]_0^1$$

$$= \left[-\frac{\pi}{2}x^4 + \pi x^2\right]_0^1 = \left[-\frac{\pi}{2}(1)^4 + \pi(1)^2\right] - \left[-\frac{\pi}{2}(0)^4 + \pi(0)^2\right]$$

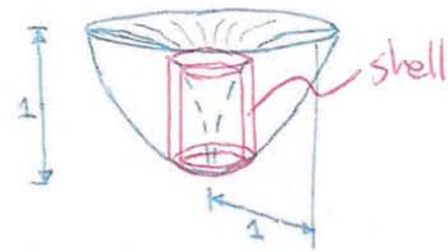
$$= -\frac{\pi}{2} + \pi = \frac{\pi}{2} \cong 1.57 \text{ units}^3$$

$$\boxed{\text{Volume} = \frac{\pi}{2} = 1.57 \text{ units}^3}$$

find the volume described in the figure



Volume revolved by $y=\sqrt{x}$ and $y=x^2$ around the y-axis?



use shell of height (y_2-y_1), radius of x and thickness dx

$$\text{Volume} = \int_0^1 (2\pi r h)\,dx = \int_0^1 2\pi(x)\big(y_2(x)-y_1(x)\big)\,dx$$

$$= \int_0^1 2\pi x\left(\sqrt{x}-x^2\right)dx = \int_0^1 2\pi x^{\frac{3}{2}} - 2\pi x^3\,dx$$

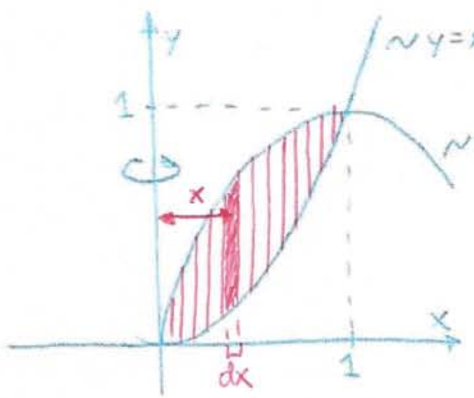
$$= \left[2\pi\left(\tfrac{2}{5}x^{\frac{5}{2}}\right) - 2\pi\left(\tfrac{1}{4}x^4\right)\right]_0^1 = \left[\frac{4\pi}{5}x^{\frac{5}{2}} - \frac{\pi}{2}x^4\right]_0^1$$

$$= \left[\frac{4\pi}{5}(1)^{\frac{5}{2}} - \frac{\pi}{2}(1)^4\right] - \left[\frac{4\pi}{5}(0)^{\frac{5}{2}} - \frac{\pi}{2}(0)^4\right]$$

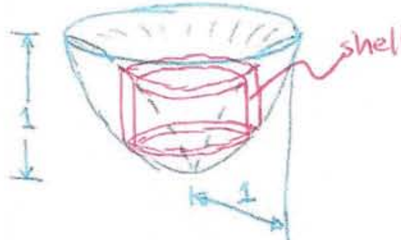
$$= \frac{4\pi}{5} - \frac{\pi}{2} = \frac{3\pi}{10} \approx 0.942 \text{ units}^3$$

$$\boxed{\text{volume} = 0.942 \text{ units}^3}$$

find the volume described in the figure



volume revolved by $y=\sin\left(\frac{\pi x}{2}\right)$ and $y=x^2$ around the y-axis?



use shells of height (y_2-y_1), radius of x and thickness dx.

$$\text{Volume} = \int_0^1 (2\pi r h)\,dx = \int_0^1 2\pi(x)\big(y_2(x)-y_1(x)\big)\,dx$$

$$= \int_0^1 2\pi x\left(\sin\left(\tfrac{\pi x}{2}\right) - x^2\right)dx = \int_0^1 2\pi x\sin\left(\tfrac{\pi x}{2}\right) - 2\pi x^3\,dx$$

$$= \int_0^1 2\pi x\sin\left(\tfrac{\pi x}{2}\right)dx - \int_0^1 2\pi x^3\,dx$$

use by parts

$u = 2\pi x \qquad dv = \sin\left(\frac{\pi}{2}x\right)dx$

$\frac{du}{dx} = 2\pi \qquad \int dv = \int \sin\left(\frac{\pi x}{2}\right)dx$

$du = 2\pi dx \qquad v = -\frac{2}{\pi}\cos\left(\frac{\pi x}{2}\right)$

$$= \left[2\pi\left(\tfrac{1}{4}x^4\right)\right]_0^1$$

$$= \left[(2\pi x)\left(-\tfrac{2}{\pi}\cos\left(\tfrac{\pi x}{2}\right)\right)\right]_0^1 - \int_0^1\left(-\tfrac{2}{\pi}\cos\left(\tfrac{\pi x}{2}\right)\right)(2\pi dx) - \left[2\pi\left(\tfrac{1}{4}x^4\right)\right]_0^1$$

$$\text{Volume} = \left[(2\pi x)\left(-\frac{2}{\pi}\cos\left(\frac{\pi x}{2}\right)\right)\right]_0^1 - \int_0^1 \left(-\frac{2}{\pi}\cos\left(\frac{\pi x}{2}\right)\right)(2\pi\,dx) - \left[2\pi\left(\frac{1}{4}x^4\right)\right]_0^1$$

$$= \left[-4x\cos\left(\frac{\pi x}{2}\right)\right]_0^1 - \left[-\frac{2}{\pi}\left(\frac{2}{\pi}\sin\left(\frac{\pi x}{2}\right)\right)(2\pi)\right]_0^1 - \left[\frac{\pi}{2}x^4\right]_0^1$$

$$= \left(\left[-4(1)\cos\left(\frac{\pi(1)}{2}\right)\right] - \left[-\frac{8}{\pi}\sin\left(\frac{\pi(1)}{2}\right)\right] - \left[\frac{\pi}{2}(1)^4\right]\right) -$$

$$\left(\left[-4(0)\cos\left(\frac{\pi(0)}{2}\right)\right] - \left[-\frac{8}{\pi}\sin\left(\frac{\pi(0)}{2}\right)\right] - \left[\frac{\pi}{2}(0)^4\right]\right)$$

$$= \left([0] - \left[-\frac{8}{\pi}\right] - \left[\frac{\pi}{2}\right]\right) - \left([0] - [0] - [0]\right)$$

$$= \frac{8}{\pi} - \frac{\pi}{2} \cong 0.976 \text{ units}^3$$

$$\boxed{\text{Volume} = \frac{8}{\pi} - \frac{\pi}{2} = 0.976 \text{ units}^3}$$