

Resonance:

Occurs when a system responds extra strongly to a periodic driving force because the total impedance becomes purely real. At this frequency, the inductor and capacitor exchange energy back and forth perfectly, and their reactive impedances exactly cancel.

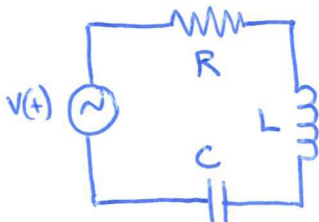
$$\begin{aligned}\text{total impedance} = Z &= Z_{\text{real}} + i \cancel{Z_{\text{imag}}}^0 \text{ (at resonance)} \\ &= R + i \cancel{X}^0 \text{ (at resonance)}\end{aligned}$$

so

$$\begin{aligned}|Z| &= \sqrt{R^2 + 0^2} = R \\ \phi_Z &= \tan^{-1}\left(\frac{0}{R}\right) = 0\end{aligned}$$

example:

for a series RLC



$$Z_{\text{total}} = R + i \left(\omega L - \frac{1}{\omega C} \right)$$

must be zero for resonance

set:

$$\omega L - \frac{1}{\omega C} = 0$$

$$\omega L = \frac{1}{\omega C}$$

$$\omega^2 LC = 1$$

$$\omega^2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

resonance frequency