Root Mean Squared

$$V_{RMS} = \underbrace{\sqrt{\underbrace{\frac{1}{T}\underbrace{\int_0^T \underbrace{v^2(t)}_{\text{squared}}\,dt}_{\text{sum}}}_{\text{average (mean)}}}}_{\text{root}}$$

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if $v(t) = V_p \sin(\omega t)$

then:

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T V_p^2 \sin^2(\omega t)\, dt}$$

and:

$$\underbrace{T}_{\text{period}} = \frac{2\pi}{\omega} \left[\frac{s}{\cancel{rads}}\right]\left[\frac{\cancel{rads}}{cycle}\right]$$

trig identity:

$$\sin^2\theta = \frac{1-\cos(2\theta)}{2}$$

$$= V_p\sqrt{\frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} \frac{1-\cos(2\omega t)}{2}\, dt}$$

$$= V_p\sqrt{\frac{\omega}{4\pi}\int_0^{\frac{2\pi}{\omega}} 1 - \cos(2\omega t)\, dt}$$

$$= V_p\sqrt{\frac{\omega}{4\pi}\left[t - \frac{1}{2\omega}\sin(2\omega t)\right]_0^{\frac{2\pi}{\omega}}}$$

$$= V_p\sqrt{\frac{\omega}{4\pi}\left[\frac{2\pi}{\omega} - \frac{1}{2\omega}\cancelto{0}{\sin\left(2\omega\frac{2\pi}{\omega}\right)} - 0 + \frac{1}{2\omega}\cancelto{0}{\sin(2\omega\cdot 0)}\right]}$$

$$= V_p\sqrt{\frac{\cancel{\omega}}{4\cancel{\pi}}\left[\frac{2\cancel{\pi}}{\cancel{\omega}}\right]}$$

$$= V_p\sqrt{\frac{1}{2}}$$

so for a sine wave

$$\boxed{V_{RMS} = \frac{V_p}{\sqrt{2}}}$$