

General 2nd Order Ordinary Differential Equation

$$a \frac{d^2 y}{dt^2} + b \frac{dy}{dt} + cy = F(t) \quad (i)$$

solve for highest order derivative ($\frac{d^2 y}{dt^2}$) in (i)

$$\frac{d^2 y}{dt^2} = \frac{1}{a} \left[F(t) - b \frac{dy}{dt} - cy \right] \quad (1a)$$

introduce 2 variables:

let:

$$x_1 = y$$

$$x_2 = \frac{dy}{dt}$$

differentiate:

$$\frac{dx_1}{dt} = x_2$$

$$\frac{dx_2}{dt} = \frac{1}{a} \left[F(t) - bx_2 - cx_1 \right] \quad (1b)$$

equ. (1a) replacing values with x's

setup calculation table:

index	time	$F(t)$	x_1	x_2	$\frac{dx_2}{dt}$
1	t_0	use	x_{10}	x_{20}	equ. (1b)
2	$t_0 + \Delta t$	$F(t) = A \sin(\omega t)$	Numerical Method		...
3	$t_0 + 2\Delta t$				
...	...				

initial conditions

General 2nd Order Ordinary Differential Equation Numerical Solution

Variables

a1s2

b2s

c5

Initial Conditions

t_o0s

y_o-0.9units

dy/dt_o0units/s

<- non-zero initial position

Sine Forcing Function

amplitude0units

frequency10Hz = cycles/s

omega = 62.83rads/s (omega = 2*pi*frequency)

Period = 0.10s (Period = 1/frequency)

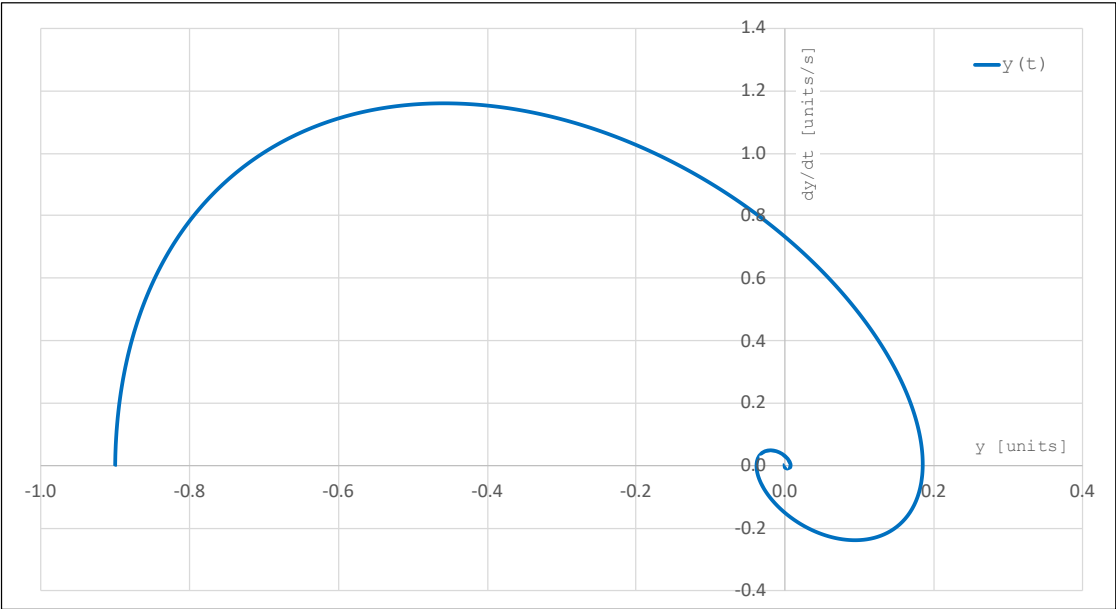
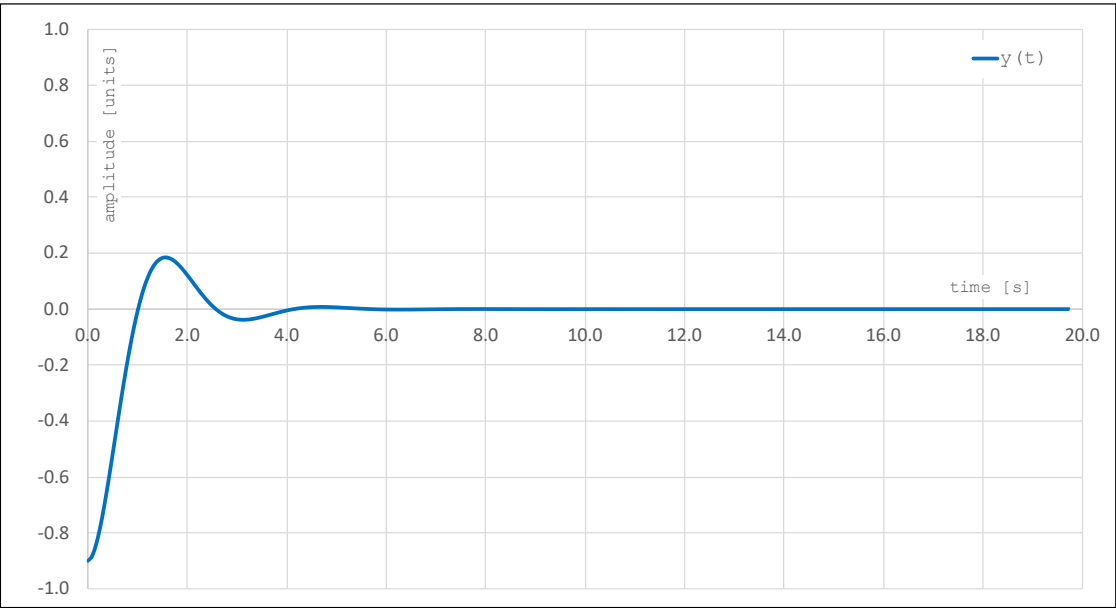
<- zero forcing function

Numerical Parameters

dt = 0.0100s

t [s]	F(t) [units]	x_1 (y) [units]	x_2 (dy/dt) [units/s]	dx_2/dt (d2y/dt2) [units/s2]
0.0000	0.0000	-0.9000	0.0000	4.5000
0.0100	0.0000	-0.8996	0.0450	4.4078
0.0200	0.0000	-0.8987	0.0891	4.3151
0.0300	0.0000	-0.8973	0.1322	4.2222
0.0400	0.0000	-0.8956	0.1745	4.1291
0.0500	0.0000	-0.8934	0.2157	4.0357
0.0600	0.0000	-0.8909	0.2561	3.9422
0.0700	0.0000	-0.8879	0.2955	3.8486
0.0800	0.0000	-0.8846	0.3340	3.7549
0.0900	0.0000	-0.8809	0.3716	3.6612
0.1000	0.0000	-0.8768	0.4082	3.5676
0.1100	0.0000	-0.8723	0.4438	3.4740
0.1200	0.0000	-0.8676	0.4786	3.3806
0.1300	0.0000	-0.8624	0.5124	3.2874
0.1400	0.0000	-0.8570	0.5453	3.1944
0.1500	0.0000	-0.8512	0.5772	3.1016
0.1600	0.0000	-0.8451	0.6082	3.0092
0.1700	0.0000	-0.8387	0.6383	2.9171
0.1800	0.0000	-0.8321	0.6675	2.8254
0.1900	0.0000	-0.8251	0.6957	2.7341
0.2000	0.0000	-0.8179	0.7231	2.6432
0.2100	0.0000	-0.8104	0.7495	2.5529
0.2200	0.0000	-0.8026	0.7750	2.4631
0.2300	0.0000	-0.7946	0.7997	2.3738
0.2400	0.0000	-0.7864	0.8234	2.2852
0.2500	0.0000	-0.7779	0.8463	2.1972
0.2600	0.0000	-0.7693	0.8682	2.1098
0.2700	0.0000	-0.7604	0.8893	2.0232
0.2800	0.0000	-0.7513	0.9096	1.9372
0.2900	0.0000	-0.7420	0.9289	1.8520
0.3000	0.0000	-0.7325	0.9475	1.7676
0.3100	0.0000	-0.7229	0.9651	1.6840
0.3200	0.0000	-0.7130	0.9820	1.6012
0.3300	0.0000	-0.7031	0.9980	1.5193
0.3400	0.0000	-0.6929	1.0132	1.4383
0.3500	0.0000	-0.6826	1.0276	1.3581
0.3600	0.0000	-0.6722	1.0411	1.2789
0.3700	0.0000	-0.6617	1.0539	1.2006

simulates a step response



General 2nd Order Ordinary Differential Equation Numerical Solution

Variables

a	4	s2
b	1	s
c	5	

Initial Conditions

t_o	0	s
y_o	0	units
dy/dt_o	0	units/s

Sine Forcing Function

amplitude	1	units
frequency	0.2	Hz = cycles/s
omega =	1.26	rads/s (omega = 2*pi*frequency)
Period =	5.00	s (Period = 1/frequency)

Numerical Parameters

dt	=	0.1000	s
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Numerical Solution

time [s]	F(t) [units]	x_1 (y) [units]	x_2 (dy/dt) [units/s]	dx_2/dt (d2y/dt2) [units/s2]
0.0000	0.0000	0.0000	0.0000	0.0000
0.1000	0.1253	0.0000	0.0000	0.0313
0.2000	0.2487	0.0003	0.0031	0.0610
0.3000	0.3681	0.0012	0.0092	0.0882
0.4000	0.4818	0.0030	0.0181	0.1121
0.5000	0.5878	0.0060	0.0293	0.1322
0.6000	0.6845	0.0102	0.0425	0.1477
0.7000	0.7705	0.0159	0.0573	0.1584
0.8000	0.8443	0.0233	0.0731	0.1637
0.9000	0.9048	0.0322	0.0895	0.1636
1.0000	0.9511	0.0428	0.1058	0.1578
1.1000	0.9823	0.0549	0.1216	0.1465
1.2000	0.9980	0.0686	0.1363	0.1297
1.3000	0.9980	0.0835	0.1492	0.1078
1.4000	0.9823	0.0995	0.1600	0.0812
1.5000	0.9511	0.1163	0.1681	0.0503
1.6000	0.9048	0.1336	0.1732	0.0159
1.7000	0.8443	0.1511	0.1748	-0.0215
1.8000	0.7705	0.1684	0.1726	-0.0610
1.9000	0.6845	0.1850	0.1665	-0.1018
2.0000	0.5878	0.2006	0.1563	-0.1429
2.1000	0.4818	0.2149	0.1420	-0.1836
2.2000	0.3681	0.2272	0.1237	-0.2229
2.3000	0.2487	0.2374	0.1014	-0.2599
2.4000	0.1253	0.2449	0.0754	-0.2936
2.5000	0.0000	0.2495	0.0460	-0.3234
2.6000	-0.1253	0.2509	0.0137	-0.3483
2.7000	-0.2487	0.2488	-0.0211	-0.3678
2.8000	-0.3681	0.2430	-0.0579	-0.3813
2.9000	-0.4818	0.2334	-0.0960	-0.3881
3.0000	-0.5878	0.2199	-0.1349	-0.3881
3.1000	-0.6845	0.2025	-0.1737	-0.3809
3.2000	-0.7705	0.1813	-0.2118	-0.3664
3.3000	-0.8443	0.1565	-0.2484	-0.3446
3.4000	-0.9048	0.1282	-0.2828	-0.3158
3.5000	-0.9511	0.0968	-0.3144	-0.2801
3.6000	-0.9823	0.0625	-0.3424	-0.2381
3.7000	-0.9980	0.0259	-0.3662	-0.1903

