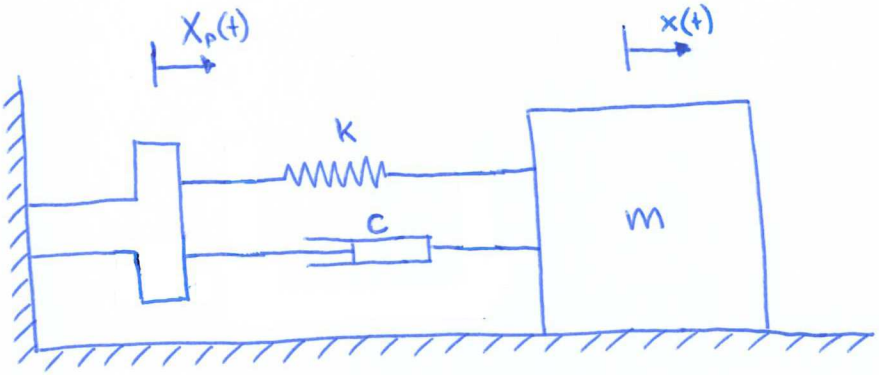
Mass Damped Spring Differential Equation



Newton's 2nd Law on mass:

$$\sum F_x = m\frac{d^2x}{dt^2} = -k(x - X_p) - c\left(\frac{dx}{dt} - \frac{dX_p}{dt}\right)$$

$$m\frac{d^2x}{dt} + c\frac{dx}{dt} + kx = \underbrace{c\frac{dX_p}{dt} + kX_p}_{F(t)} \qquad (1)$$

solve for highest order derivative $\left(\frac{d^2x}{dt^2}\right)$ in (1)

$$\frac{d^2x}{dt^2} = \frac{1}{m}\left[F(t) - c\frac{dx}{dt} - kx\right] \qquad (1a)$$

introduce 2 variables:

let

$$x_1 = x$$

$$x_2 = \frac{dx}{dt}$$

differentiate:

$$\frac{dx_1}{dt} = x_2$$

$$\frac{dx_2}{dt} = \frac{1}{m}\left[F(t) - cx_2 - kx_1\right] \qquad (1b)$$

equ (1a) replacing variables with state variables (x's)

setup calculation table:

index	time	X_p	$\frac{dX_p}{dt}$	$F(t)$	x_1	x_2	$\frac{dx_2}{dt}$
							equ (1b)

use: $X_p = A\sin(\omega t)$

so: $\frac{dX_p}{dt} = A\omega\cos(\omega t)$

From equ (1):

$$F(t) = c\frac{dX_p}{dt} + kX_p$$

Mass-Damper-Spring Differential Equation Numerical Soluton

Variables

m	10	kg
c	2	kg/s = Ns/m
k	20	kg/s2 = N/m

Initial Conditions

t_o	0	s	
x_o	-0.9	m	<- non-zero initial position
dx/dt_o	0	m/s	

Sine Displacement (forcing function)

amplitude	0	m	<- zero amplitude forcing function
frequency	10	Hz = cycles/s	
omega =	62.83	rads/s	(omega = 2*pi*frequency)
Period =	0.10	s	(Period = 1/frequency)

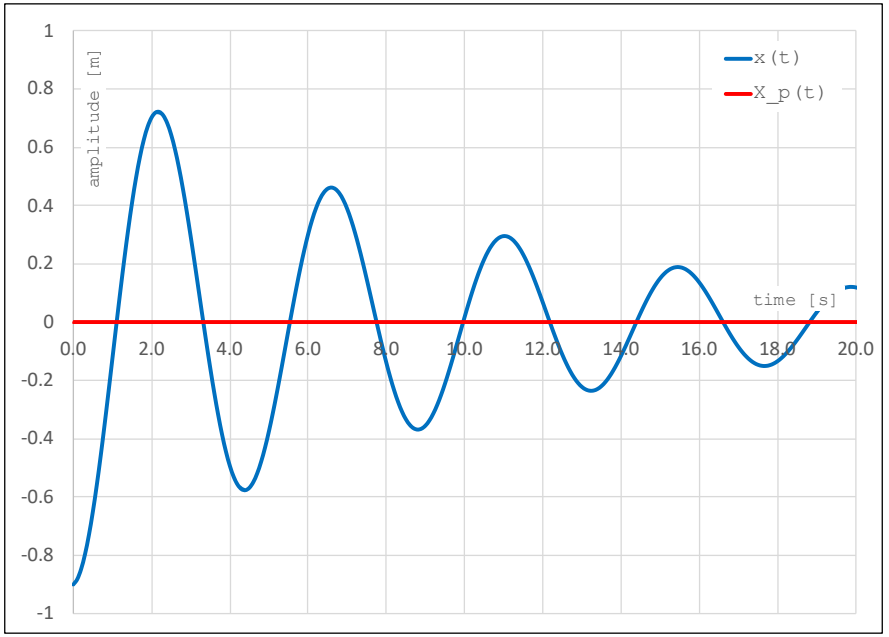
simulates a step response

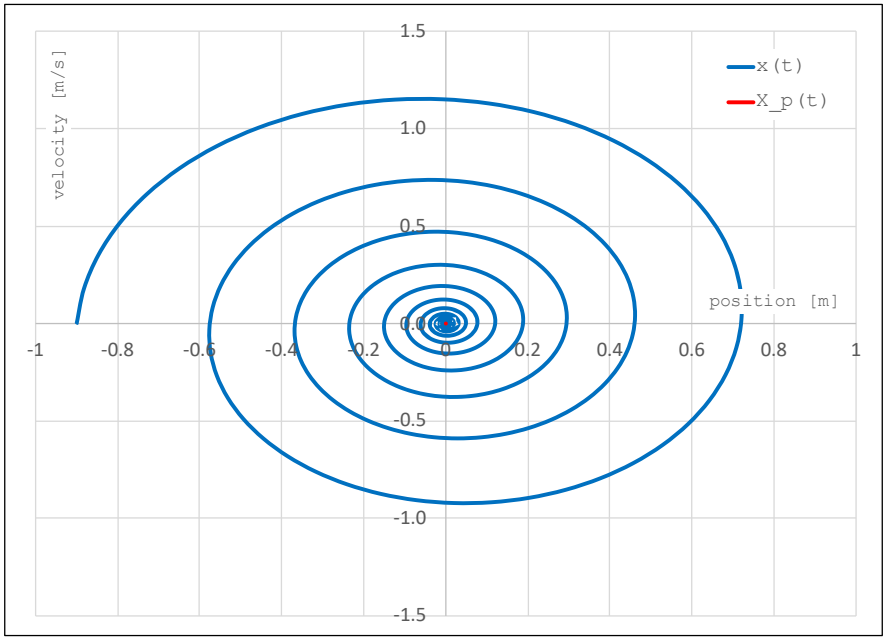
Numerical Parameters

dt	= 0.1000	s	

Numerical Solution

t [s]	X_p(t) [m]	dX_p/dt [m/s]	F(t) [N]	x_1 (x) [m]	x_2 (dx/dt) [m/s]	dx_2/dt (d2x/dt2) [m/s2]
0.0000	0.0000	0.0000	0.0000	-0.9000	0.0000	1.8000
0.1000	0.0000	0.0000	0.0000	-0.8820	0.1800	1.7280
0.2000	0.0000	0.0000	0.0000	-0.8467	0.3528	1.6229
0.3000	0.0000	0.0000	0.0000	-0.7952	0.5151	1.4874
0.4000	0.0000	0.0000	0.0000	-0.7288	0.6638	1.3249
0.5000	0.0000	0.0000	0.0000	-0.6492	0.7963	1.1391
0.6000	0.0000	0.0000	0.0000	-0.5582	0.9102	0.9343
0.7000	0.0000	0.0000	0.0000	-0.4578	1.0037	0.7149
0.8000	0.0000	0.0000	0.0000	-0.3503	1.0751	0.4856
0.9000	0.0000	0.0000	0.0000	-0.2379	1.1237	0.2511
1.0000	0.0000	0.0000	0.0000	-0.1230	1.1488	0.0163
1.1000	0.0000	0.0000	0.0000	-0.0080	1.1504	-0.2141
1.2000	0.0000	0.0000	0.0000	0.1049	1.1290	-0.4356
1.3000	0.0000	0.0000	0.0000	0.2135	1.0855	-0.6440
1.4000	0.0000	0.0000	0.0000	0.3156	1.0211	-0.8353
1.5000	0.0000	0.0000	0.0000	0.4093	0.9375	-1.0061
1.6000	0.0000	0.0000	0.0000	0.4930	0.8369	-1.1534
1.7000	0.0000	0.0000	0.0000	0.5652	0.7216	-1.2747
1.8000	0.0000	0.0000	0.0000	0.6246	0.5941	-1.3680
1.9000	0.0000	0.0000	0.0000	0.6703	0.4573	-1.4321
2.0000	0.0000	0.0000	0.0000	0.7017	0.3141	-1.4663
2.1000	0.0000	0.0000	0.0000	0.7185	0.1675	-1.4704
2.2000	0.0000	0.0000	0.0000	0.7205	0.0204	-1.4451
2.3000	0.0000	0.0000	0.0000	0.7081	-0.1241	-1.3914
2.4000	0.0000	0.0000	0.0000	0.6818	-0.2632	-1.3109
2.5000	0.0000	0.0000	0.0000	0.6424	-0.3943	-1.2059
2.6000	0.0000	0.0000	0.0000	0.5909	-0.5149	-1.0788
2.7000	0.0000	0.0000	0.0000	0.5286	-0.6228	-0.9326
2.8000	0.0000	0.0000	0.0000	0.4570	-0.7160	-0.7708
2.9000	0.0000	0.0000	0.0000	0.3777	-0.7931	-0.5967
3.0000	0.0000	0.0000	0.0000	0.2924	-0.8528	-0.4142
3.1000	0.0000	0.0000	0.0000	0.2030	-0.8942	-0.2271
3.2000	0.0000	0.0000	0.0000	0.1113	-0.9169	-0.0392
3.3000	0.0000	0.0000	0.0000	0.0192	-0.9208	0.1458
3.4000	0.0000	0.0000	0.0000	-0.0714	-0.9063	0.3241
3.5000	0.0000	0.0000	0.0000	-0.1588	-0.8739	0.4924
3.6000	0.0000	0.0000	0.0000	-0.2413	-0.8246	0.6475
3.7000	0.0000	0.0000	0.0000	-0.3173	-0.7599	0.7865





Mass-Damper-Spring Differential Equation Numerical Soluton

Variables

m	50	kg
c	10	kg/s = Ns/m
k	20	kg/s2 = N/m

Initial Conditions

t_o	0	s
x_o	0	m
dx/dt_o	0	m/s

Sine Displacement (forcing function)

amplitude	0.9	m
frequency	0.2	Hz = cycles/s
omega =	1.26	rads/s (omega = 2*pi*frequency)
Period =	5.00	s (Period = 1/frequency)

Numerical Parameters

dt = 0.0100 s

Numerical Solution

t [s]	X_p(t) [m]	dX_p/dt [m/s]	F(t) [N]	x_1 (x) [m]	x_2 (dx/dt) [m/s]	dx_2/dt (d2x/dt2) [m/s2]
0.0000	0.0000	1.1310	11.3097	0.0000	0.0000	0.2262
0.0100	0.0113	1.1309	11.5350	0.0000	0.0023	0.2302
0.0200	0.0226	1.1306	11.7585	0.0001	0.0046	0.2342
0.0300	0.0339	1.1302	11.9801	0.0001	0.0069	0.2382
0.0400	0.0452	1.1295	12.1998	0.0002	0.0093	0.2420
0.0500	0.0565	1.1287	12.4176	0.0003	0.0117	0.2459
0.0600	0.0678	1.1278	12.6335	0.0005	0.0142	0.2496
0.0700	0.0791	1.1266	12.8473	0.0007	0.0167	0.2534
0.0800	0.0903	1.1253	13.0591	0.0008	0.0192	0.2570
0.0900	0.1016	1.1237	13.2689	0.0011	0.0218	0.2606
0.1000	0.1128	1.1221	13.4766	0.0013	0.0244	0.2641
0.1100	0.1240	1.1202	13.6821	0.0016	0.0270	0.2676
0.1200	0.1352	1.1181	13.8854	0.0019	0.0297	0.2710
0.1300	0.1464	1.1159	14.0866	0.0022	0.0324	0.2744
0.1400	0.1575	1.1135	14.2856	0.0026	0.0351	0.2777
0.1500	0.1686	1.1109	14.4823	0.0029	0.0379	0.2809
0.1600	0.1797	1.1082	14.6767	0.0033	0.0407	0.2841
0.1700	0.1908	1.1053	14.8688	0.0038	0.0436	0.2872
0.1800	0.2018	1.1022	15.0585	0.0042	0.0464	0.2902
0.1900	0.2128	1.0989	15.2459	0.0047	0.0493	0.2932
0.2000	0.2238	1.0954	15.4308	0.0053	0.0523	0.2961
0.2100	0.2348	1.0918	15.6134	0.0058	0.0552	0.2989
0.2200	0.2457	1.0880	15.7934	0.0064	0.0582	0.3017
0.2300	0.2565	1.0841	15.9710	0.0070	0.0612	0.3044
0.2400	0.2673	1.0799	16.1460	0.0076	0.0643	0.3070
0.2500	0.2781	1.0756	16.3185	0.0083	0.0674	0.3096
0.2600	0.2888	1.0711	16.4884	0.0090	0.0705	0.3121
0.2700	0.2995	1.0665	16.6557	0.0098	0.0736	0.3145
0.2800	0.3102	1.0617	16.8204	0.0105	0.0767	0.3169
0.2900	0.3208	1.0567	16.9824	0.0113	0.0799	0.3191
0.3000	0.3313	1.0516	17.1418	0.0122	0.0831	0.3214
0.3100	0.3418	1.0462	17.2984	0.0130	0.0863	0.3235
0.3200	0.3522	1.0408	17.4523	0.0139	0.0895	0.3256
0.3300	0.3626	1.0351	17.6034	0.0148	0.0928	0.3276
0.3400	0.3729	1.0293	17.7518	0.0158	0.0961	0.3295
0.3500	0.3832	1.0233	17.8974	0.0168	0.0994	0.3314
0.3600	0.3934	1.0172	18.0401	0.0178	0.1027	0.3331
0.3700	0.4035	1.0109	18.1800	0.0189	0.1060	0.3348