Quadratic Formula

2nd order polynomial (the quadratic equation)

$$y = ax^2 + bx + c$$

Complete the square

$$y = ax^2 + bx + c$$

$$= a\left[x^2 + \frac{b}{a}x \quad\right] + c$$

$$= a\left[x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right] + c$$

$$= a\left[\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right] + c$$

$$= a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$$

$$\boxed{y = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}}$$

Solve for x

$$y = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$$

$$\frac{b^2}{4a} - c + y = a\left(x + \frac{b}{2a}\right)^2$$

$$\frac{b^2}{4a^2} - \frac{c}{a} + \frac{y}{a} = \left(x + \frac{b}{2a}\right)^2$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a} + \frac{y}{a}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2}{4a^2} - \frac{c}{a} + \frac{y}{a}}$$

$$x = \pm\sqrt{\frac{b^2}{4a^2} - \frac{c}{a} + \frac{y}{a}} - \frac{b}{2a}$$

$$x = \pm\sqrt{\left(\frac{1}{4a^2}\right)\left(b^2 - 4ac + 4ay\right)} - \frac{b}{2a}$$

$$x = \pm\sqrt{\left(\frac{1}{2a}\right)^2\left(b^2 - 4ac + 4ay\right)} - \frac{b}{2a}$$

$$x = \frac{\pm\sqrt{b^2 - 4ac + 4ay}}{2a} - \frac{b}{2a}$$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac + 4ay}}{2a}}$$

For $y = 0$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Complete the square

2nd order polynomial (the quadratic equation)

$y = ax^2 + bx + c$ ...Form 1

$y = a(x + h)^2 + k$ ...Form 2

To convert from Form 2 into Form 1 => expand

To convert from Form 1 into Form 2 => complete the square

$y = ax^2 + bx + c$ ...Form 1

$= a\left[x^2 + \frac{b}{a}x \quad\right] + c$

$= a\left[x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right] + c$

$= a\left[\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right) - \left(\frac{b}{2a}\right)^2\right] + c$

$= a\left[\left(x + \frac{b}{2a}\right)^2 \quad - \left(\frac{b}{2a}\right)^2\right] + c$

$= a\left(x + \frac{b}{2a}\right)^2 - a\left(\frac{b}{2a}\right)^2 + c$

$= a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$

the term added & subtracted is equal to $\left(\frac{\text{term}}{2}\right)^2$

$y = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$ ...Form 2

$y = a(x + h)^2 + k$

$\therefore\ h = \frac{b}{2a}\ ,\ k = c - \frac{b^2}{4a}$

$y = x^2 + 4x - 2$

$= x^2 + 4x + 4 - 4 - 2$

$= (x+2)^2 - 4 - 2$

$= (x+2)^2 - 6$

$y = 2x^2 + 4x - 2$

$= 2[x^2 + 2x \quad] - 2$

$= 2[x^2 + 2x + 1 - 1] - 2$

$= 2[(x+1)^2 - 1] - 2$

$= 2(x+1)^2 - 4$

$y = x^2 + 8x + 10$

$= x^2 + 8x + 16 - 16 + 10$

$= (x+4)^2 - 16 + 10$

$= (x+4)^2 - 6$

$y = 3x^2 - 12x$

$= 3[x^2 - 4x \quad]$

$= 3[x^2 - 4x + 4 - 4]$

$= 3[(x-2)^2 - 4] = 3(x-2)^2 - 12$

$y = x^2 - 6x + 4$

$= x^2 - 6 + 9 - 9 + 4$

$= (x-3)^2 - 9 + 4$

$= (x-3)^2 - 5$

$y = -2x^2 - 4x + 2$

$= -2[x^2 + 2x \quad] + 2$

$= -2[x^2 + 2x + 1 - 1] + 2$

$= -2[(x+1)^2 - 1] + 2 = 2(x+1)^2$

$y = x^2 - 7x - 13$

$= x^2 - 7x + \left(\frac{7}{2}\right)^2 - \left(\frac{7}{2}\right)^2 - 13$

$= \left(x - \frac{7}{2}\right)^2 - \left(\frac{7}{2}\right)^2 - 13$

$= \left(x - \frac{7}{2}\right)^2 - \frac{101}{4}$

$= (x - 3.5)^2 - 25.25$

$y = -x^2 + 2x + 3$

$= -[x^2 - 2x \quad] + 3$

$= -[x^2 - 2x + 1 - 1] + 3$

$= -[(x-1)^2 - 1] + 3$

$= -(x-1)^2 + 4$